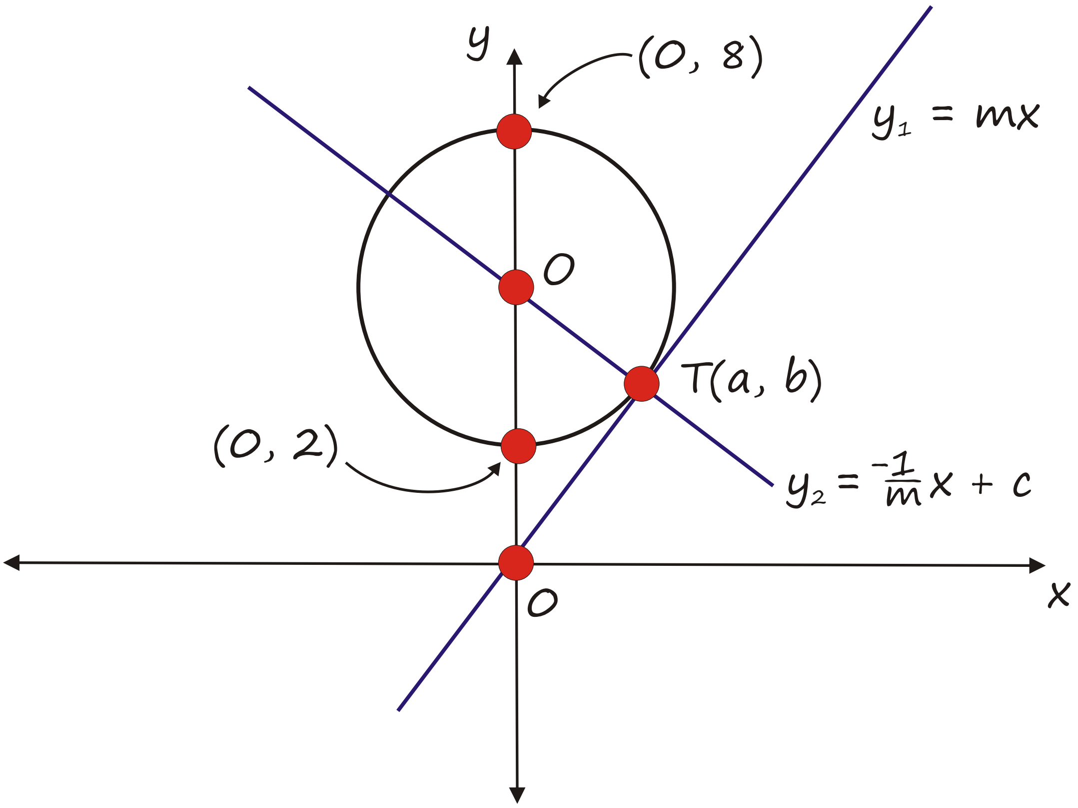
# NCEA LEVEL 2 MATHEMATICS

## 2.1 - AS91256

### Apply Coordinate Geometry Methods in Solving Problems




# Questions and Answers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**NCEA Level 2 Mathematics, Questions & Answers**
**AS91256 Apply Coordinate Geometry Methods in Solving Problems**
Kim Freeman

This edition is part of an eBook series designed to help you study towards NCEA.

**Note:** This achievement standard is assessed by each individual school and students can use any appropriate technology.

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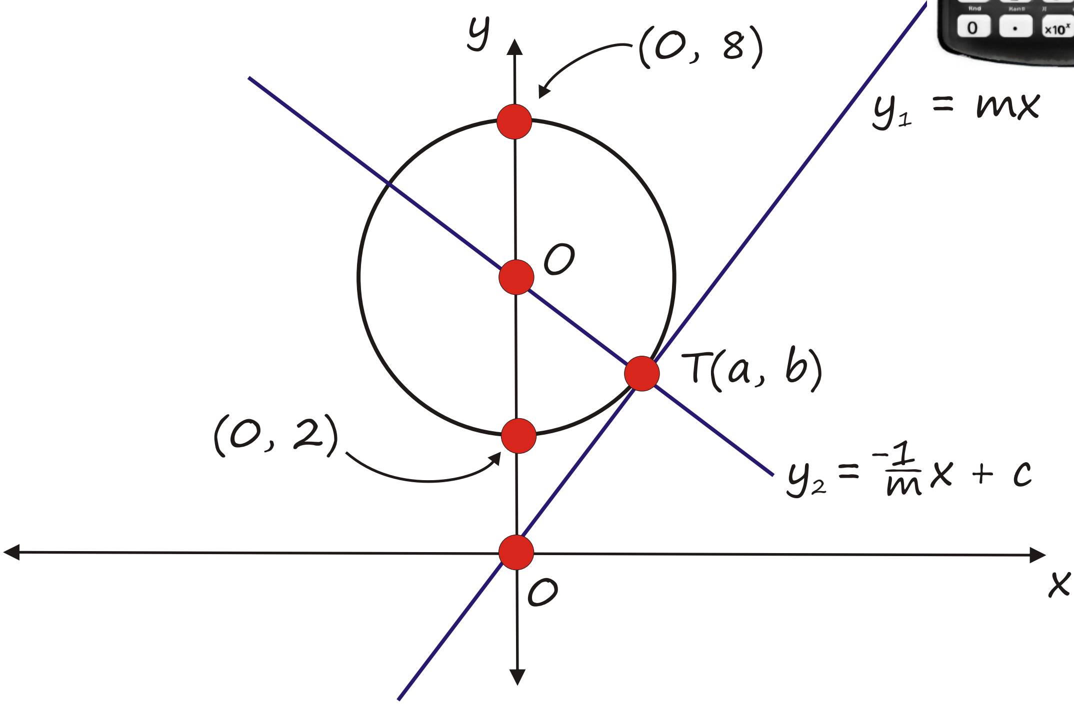
# The DS-742ET


Find the tangent point T.

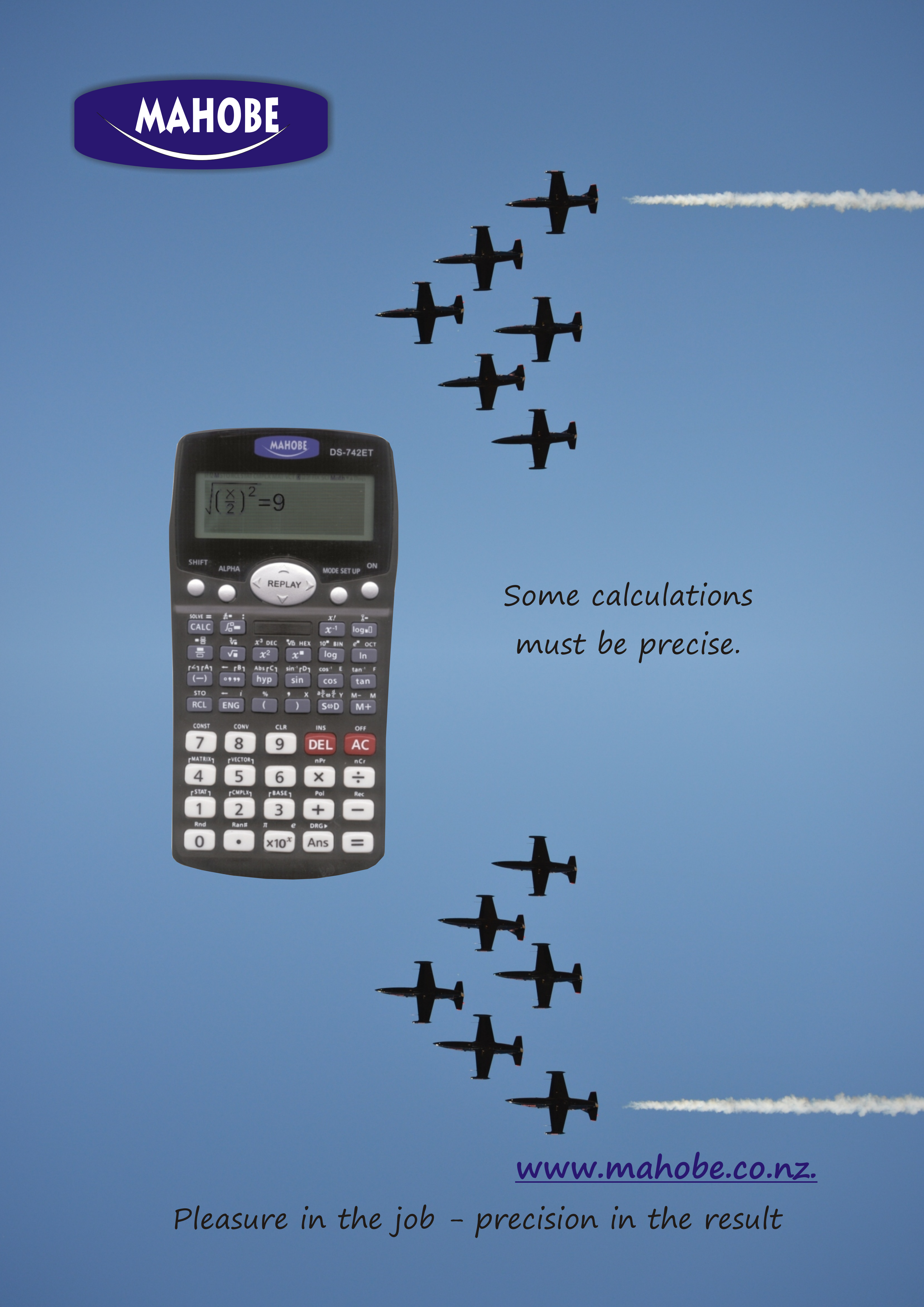
Find the equations $y_1$ and $y_2$.

Is $y_1$ perpendicular to $y_2$?




By the end of this booklet we hope you will have learnt to solve this problem.

The exercises were supplied by Kim Freeman. The booklet was funded by sales of the Mahobe DS-742ET calculator.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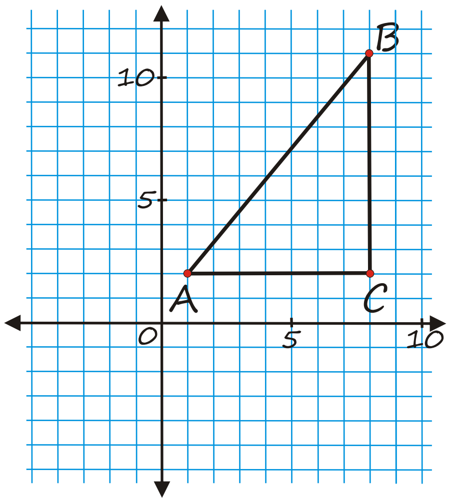
# The Distance Formula




The grid on the left shows a right angled triangle formed by joining the points A (1, 2), B(8, 12) and C(8, 2).

If the grid is a scale drawing and each square in the grid represents a side length of 1 m then find the length of AB.

*This is a simple Pythagoras problem*

$AC = 8 - 1$
$= 7$ m

$BC = 11 - 2$
$= 9$ m

$AB^2 = AC^2 + BC^2$

$AB^2 = 7^2 + 9^2$

$AB = \sqrt{130}$

$AB = 11.40$ m

*The distance theorem comes from the Pythagoras theorem.*

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Is the shape formed by joining the points A(-5, 4), B(5, 4) and C(0, -2) an isosceles, equilateral or scalene triangle?

$AB = \sqrt{(5 - (-5))^2 + (4 - 4)^2}$

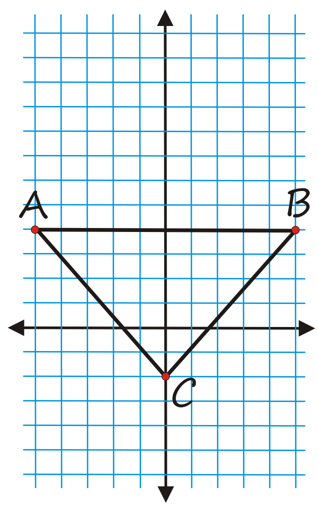
$AB = 10$

$AC = \sqrt{(0 - (-5))^2 + (-2 - 4)^2}$

$AC = 7.81$

$BC = \sqrt{(0 - 5)^2 + (-2 - 4)^2}$

$= 7.81$




*The lengths AC = BC therefore the triangle is an isosceles.*

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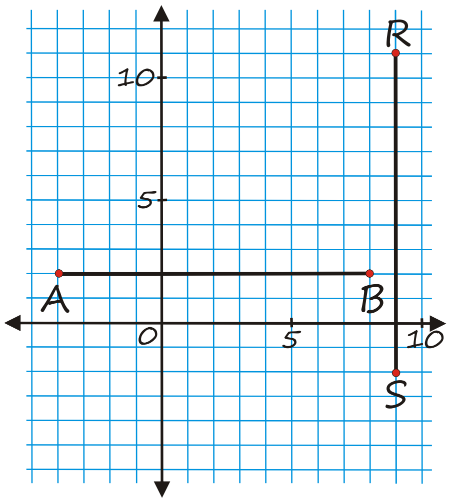
# The Mid-Point Formula




The grid on the left shows two lines. The first joins the points AB and the second joins the points RS. Find the midpoint of AB and the midpoint of RS.

To calculate the midpoints, find the lengths and divide by 2.

A = (-4, 2), B = (8, 2)

The y value must be 2

To find the x value $[(-4) + 8] \div 2 = 2$

Therefore the midpoint of AB = (2, 2)

R = (9, 11), S = (9, -2)

Because it is a vertical line, the x value must be 9.

To find the y value $[11 + (-2)] \div 2 = 4.5$

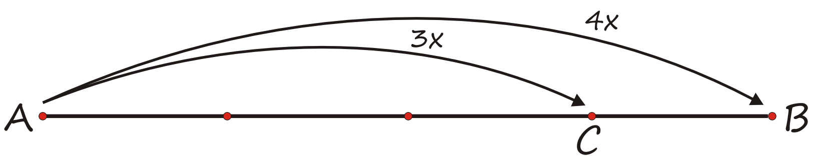
Therefore the midpoint of RS = (9, 4.5).

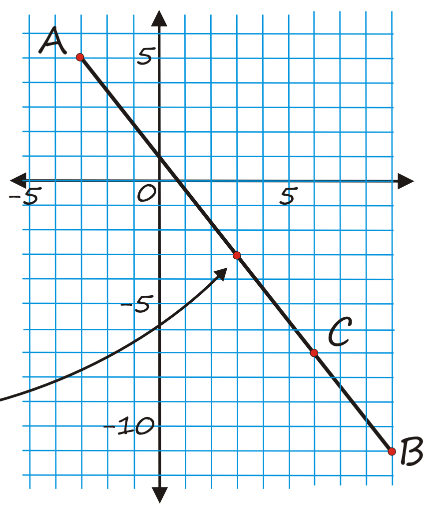
To find the midpoint of a line joined between $(x_1, y_1)$ and $(x_2, y_2)$.

Let (x, y) = the midpoint $\quad x = \dfrac{x_1 + x_2}{2} \quad y = \dfrac{y_1 + y_2}{2}$

**e.g.** The line joining A(-3, 5) and B(9, -11) can be divided so that AC:CB = 3:1. Find point C.

The line must be divided into 4 equal parts.




Using the midpoint formula:

$$\frac{[(-3) + 9)]}{2}, \frac{[5+(-11)]}{2} = (3, -3)$$

Now find C, the midpoint between (3, -3) and (9, -11)

$$= \frac{[3 + 9]}{2}, \frac{[(-3)+(-11)]}{2} \qquad C = (6, -7)$$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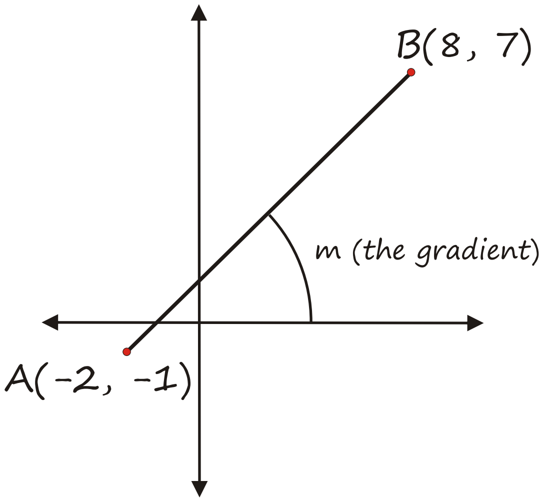
# Gradients

The gradient is best described as the slope of a line. It can also be calculated as the positive angle between the x-axis and the line.




The gradient between A and B is:

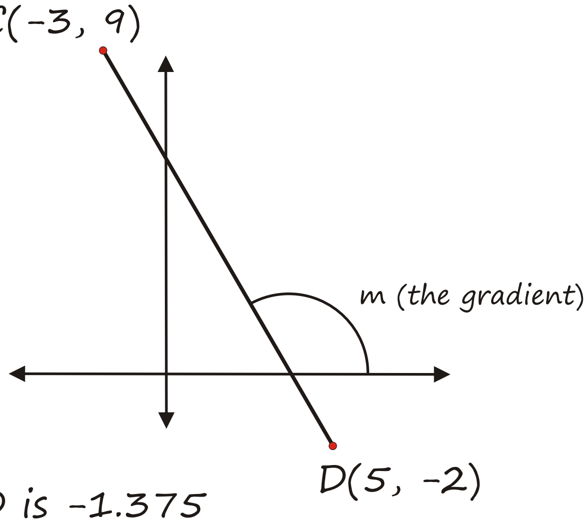
$$\frac{7 - (-1)}{8 - (-2)} = \frac{8}{10}$$

Therefore the gradient is 0.8.

The angle, $\tan^{-1}(0.8) = 38.65°$.

To find the gradient between two points use:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$




The gradient between C and D is:

$$\frac{(-2) - 9}{5 - (-3)} = \frac{-11}{8}$$

Therefore the gradient of CD is -1.375

The positive angle between CD is $\tan^{-1}(1.375) = 53.97°$

This means the two angles along the x axis are 53.97°, 126.03°

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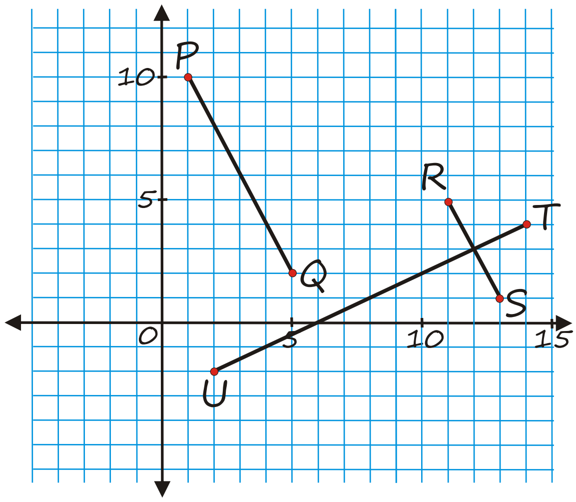
# More About Gradients

If two lines have the same gradient then they must be parallel.

**e.g.** Are lines PQ and RS parallel?
Point P = (1, 10) and Q = (5, 2)
Point R = (11, 5) and S = (13, 1).




*Gradient of PQ* $\dfrac{2-10}{5-1} = -2$

*Gradient of RS* $\dfrac{1-5}{13-11} = -2$

*The two gradients are the same.*

*Therefore the lines are parallel.*

If the product of two lines' gradients is -1 then they must be perpendicular.

**e.g.** Are lines RS and TU perpendicular?
Point R = (11, 5) and S = (13, 1).
Point T = (14, 4) and U = (2, -2).

*Gradient of RS* = -2 *(we worked this one out above)*

*Gradient of TU* $\dfrac{(-2)-4}{2-14} = \dfrac{1}{2}$

*Multiply the two gradients together* $-2 \times \dfrac{1}{2} = -1$.

*The two lines must be perpendicular.*

**e.g.** Are the points L(0, 6), M(3, -4) and N(10, -25) co-linear?

*Co-linear means that they are all in line.*

*It means that the gradient of LM = MN*

*Gradient LM* $= \dfrac{(-4)-6}{3-0} = \dfrac{-10}{3} = -3.33$

*Gradient LN* $= \dfrac{(-25)-(-4)}{10-3} = \dfrac{-21}{7} = -3$

*The gradients are different. The points are not co-linear.*



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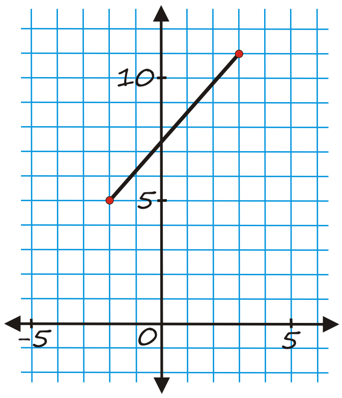
# Equation of a Line




*To find the equation of a line between two points use:* $y - y_1 = m(x - x_1)$

**e.g.** Find the equation of the line joining (-2, 5) and (3, 11).

*First find the gradient of the line.*

Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{11 - 5}{3 - (-2)}$

$m = 1.2$

*Using any one of the points e.g. (3, 11)*

$y - 11 = 1.2(x - 3)$

$y - 11 = 1.2x - 3.6$

$y = 1.2x + 7.4$

**e.g.** On a map grid a fence passes through the point (-5, 7) and follows a path perpendicular to the line y = 2x + 5. Find the equation of the fence line.

*The gradient of* $y = 2x + 5$ *is 2 (if* $y = mx + c$*, the gradient =* $m$*)*

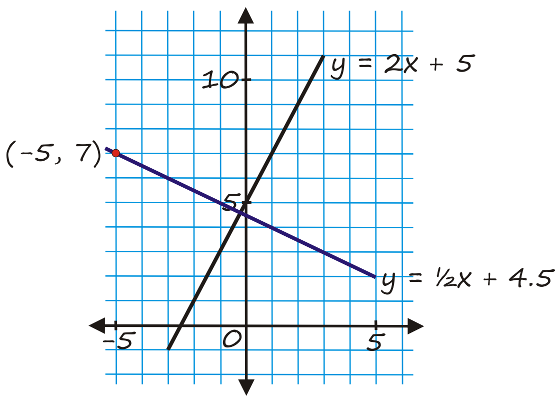
*Using* $m_1 \times m_2 = -1$*, the gradient of the perpendicular line =* $-\frac{1}{2}$

*Put all this information into:* $y - y_1 = m(x - x_1)$.

$y - 7 = -\frac{1}{2}[x - (-5)]$

$y - 7 = -\frac{1}{2}x - 2.5$

$y = -\frac{1}{2}x + 4.5$

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# Some Coordinate Geometry Examples

**e.g.** The distance between points (-4, 5) and (3, y) is $\sqrt{58}$.
What is the value of y?

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{58} = \sqrt{(3 - (-4))^2 + (y - 5)^2}$$

square each side

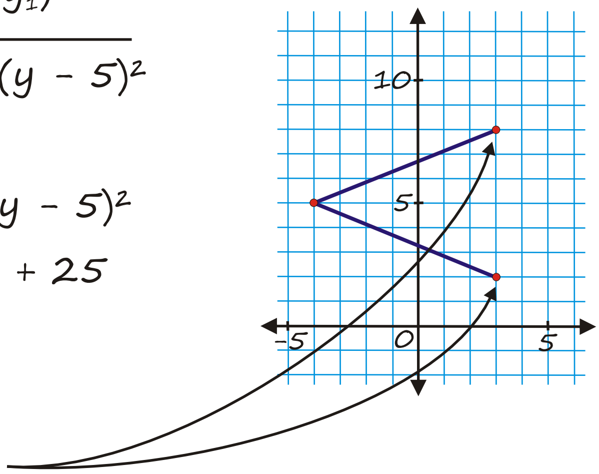
$$58 = (3 - (-4))^2 + (y - 5)^2$$

$$58 = 49 + y^2 - 10y + 25$$

$$0 = y^2 - 10y + 16$$

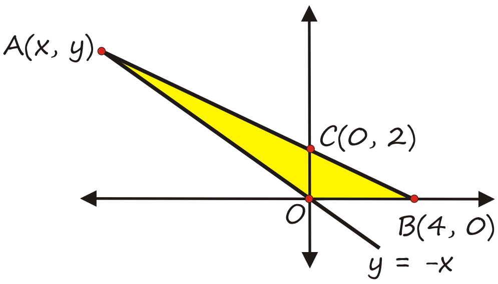
$$0 = (y - 8)(y - 2)$$

$$y = 8 \text{ or } x = 2$$




For this question you had to remember your Year 11 factorising skills. If you joined (-4, 5) with the two possible points you would get an isosceles triangle.

**e.g.** In the figure below find the area of the shaded triangle AOB.




We know the values of points C and B. Use them to get more information.

The gradient of CB = $-\frac{1}{2}$

The equation of CB using B(4, 0)

$$y - y_1 = m(x - x_1)$$

$$y = -\tfrac{1}{2}(x - 4)$$

$$y = -\tfrac{1}{2}x + 2$$

Find where $y = -x$ and $y = -\frac{1}{2}x + 2$ intersect.

$$y = y \text{ therefore } -x = -\tfrac{1}{2}x + 2$$

$$-\tfrac{1}{2}x = 2 \quad \Rightarrow x = -4$$

Using $y = -x$ means that point A is (-4, 4)

This means the height of the triangle is 4 units.

The base is also 4 units. Area is $\frac{1}{2} \times 4 \times 4$ = 8 square units



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# Exercises

**1.** F is the point (-6, 7) and G is (8, 0)

**a.** Find the midpoint of FG.

**b.** Find the length of FG.

**c.** Find the equation of the line FG.

**d.** Find the equation of the line that is parallel to FG and that passes through the point (1, 1).

**2.** Find the equation of the line that is perpendicular to $y = 2x - 5$ and that passes through the point (4, -1).

**3.** Find the equation of the line that passes through (-2, -2) and is parallel to $y = 4x + 3$.

**4.** Find the coordinates of the point of intersection of the lines $y = x - 10$ and $2x + 3y = 55$.

**5.** Find the value of k so that the points A(2, 7), B(6, k) and C(20, -20) are collinear. (**Hint** find the gradient and equation of AC first.)

**6.** Is the triangle with vertices J(-3, -1), K(0, 8) and L(3, 2) a right angled triangle?

**7.** The line $8x + 5y = 10$ is perpendicular to the line $10x + ky = 11$. Calculate the value of the constant k.

**8.** R is the point (-5, 5) and S is the point (9, -2).
Point V(k, -10) lies on the line drawn through the points R and S.
Find the value of k.

**9.** Audrey and Joss are making a kite. They start by drawing their plan onto some squared paper. The initial points that they use are O(0, 0), A(7, 24) and B(25, 0).

**a.** Show that OAB is an isosceles triangle.

**b.** Audrey says that the fourth vertex of the kite should be on a line that is through (0, 0) and perpendicular to the line AB. Find the equation of this line.

**c.** Show that the line found in b. goes through the midpoint of AB.

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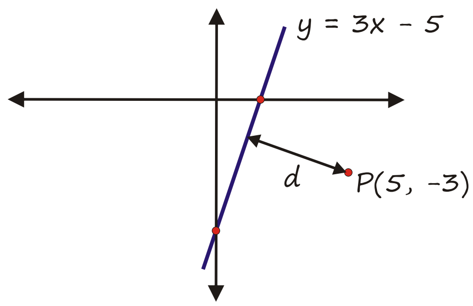
**10.** Show that the equations $x + y + 2 = 0$ and $x - y - 8 = 0$ meet at the point (3, -5). Find the equation of the horizontal bisector of these two lines.

**11.** Find the equation of the tangent to the circle $x^2 + y^2 = 10$ at the point (3, 1).

**12.** Find the shortest distance, d, between the line $y = 3x - 5$ and P(5, -3).




**13.** The distance of a point P(k, -1) and the line $y = x - 5$ is 6 units. Calculate the possible values of k.

**14.** Show that the lines $3x - 2y + 9 = 0$ and $-3x + 2y + 12 = 0$ are parallel and calculate the shortest distance between the two lines.

**15.** The quadrilateral formed by the points A(2, 2), B(5, -2), C(9, 1) and D(6, 5) is either a rhombus, parallelogram or square. By using coordinate geometry, calculate which quadrilateral it is.

**16.** Show through coordinate geometry that the points A(-2, 2), B(1, 4), C(2, 8) and D(-1, 6) can be joined to form a parallelogram.

**17.** Two garden sprinklers (S1 and S2) are joined by a water pipe under a field. According to a map of the field the grid references are S1(4, 6) and S2(16, 10). The grid units are in metres.

- **a.** What is the distance between the two sprinklers?
- **b.** The main water pipe, connects to the midpoint of the pipe between S1 and S2. Calculate the midpoint of S1 and S2.
- **c.** Show that the equation of the pipe line between S1 and S2 is $3y - x - 14 = 0$.
- **d.** The main water pipe is at right angles to the line midway between S1 and S2. Write the equation of the main water pipe line.
- **e.** The main water pipe extends beyond this connection to a point S3. The distances between S1 and S3 and S2 and S3 are both equal to 14.14 m. If the x co-ordinate is 6, find the y coordinate of S3.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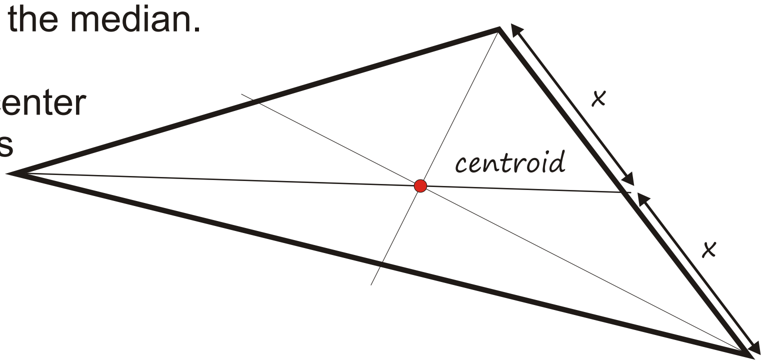
# Triangle Centres

Three types of triangle centers are shown below

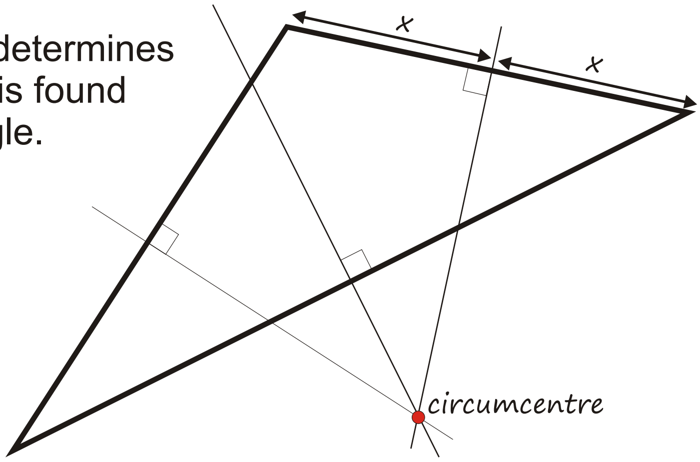
The **CENTROID**
The centroid of a triangle is constructed by taking any triangle and connecting the midpoints of each side to the opposite vertex. The line segment created by connecting these points is called the median.

The centroid of a triangle is the center of the triangle's mass. It is always found inside the triangle.

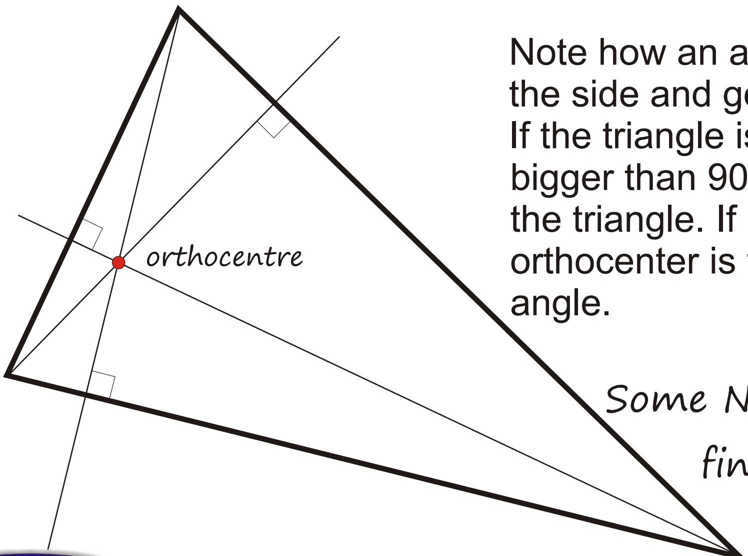



The **CIRCUMCENTRE**
The three perpendicular bisectors of the sides of a triangle meet in one point called the circumcenter. It is also the center of a circle that can be drawn where all three vertices will be found on the circumference.

The shape of the triangle determines whether the circumcentre is found inside or outside the triangle.




The **ORTHOCENTRE**
The three altitudes of a triangle meet in one point called the orthocenter.




Note how an altitude is perpendicular (90°) to the side and goes through the opposite vertex. If the triangle is obtuse (i.e. has one angle bigger than 90°, then the orthocenter is outside the triangle. If it is a right triangle, the orthocenter is the vertex which is the right angle.

*Some NCEA questions will involve finding triangle centres.*

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# Exercises

**1.** AB is the diameter of a circle. A is at point (-15, 2) and B is (7, 12). What are the coordinates of the circle centre?

**2.** A circle has a centre of (8, 1). FG is the diameter of the circle. If F is (2, -3) what are the coordinates of point G?

**3.** Jordan's golf ball is 25 metres short and 8 metres to the right of the hole. On his next shot the ball lands 1 metre to the left and 3 metres beyond the hole. If the ball went in a straight line, how far did it travel?

**4.** The distance between (7, 5) and (x, -3) is 10 units. Find x, the missing coordinate.

**5.** RS is a chord of a circle with centre O(2, 1). R is (-4, 9) and S is (10, 7). A line, L, is perpendicular to RS and bisects it. Does the line L pass through the centre of the circle?

**6.** On a map grid two locations, x and y, are 10 cm apart. The point x is at (2, 1) and y is at the point (-6, a). What are the possible values of a?

**7.** Find the point on the y axis which is equidistant to the points M(-4, 0) and N(9, 5).

**8.** A triangle ABC has vertices A(-2, -2), B(1, 8) and C(6, 2). If the points D and E are the midpoints of AB and AC, show that ED = ½BC.

**9.** The line segment MN has endpoints M(-2, 3) and N(6, 19). Find the coordinates of point P which divides MN in the ratio MP: PN = 3:1.

**10.** Find the area of the triangle with vertices X(-2, 4), Y(3, 2) and Z(1, -3).

**11.** Show that the points F(4, 1), G(5, -2) and H(6, -5) are co-linear (all in line).

**12.** Show that the points P(1, 2), Q(2, -1) and R(-4, -3) form a right angled triangle.

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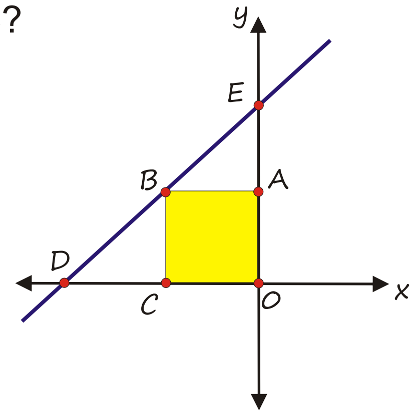
**13.** Find the equation of the line parallel to the line joining the points B(-1, 4) and C(-5, -10) and which intercepts the x axis at (5, 0).

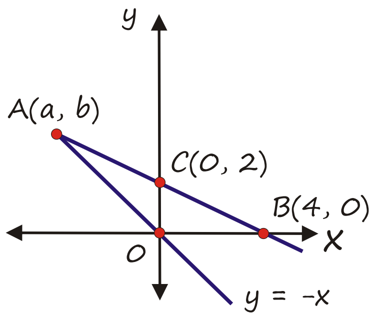
**14.** In the figure shown the equation of the line DE is $y = \sqrt{3}x + 3\sqrt{3} + 3$.

What is the area of the square OABC?




**15.** The lines y = 2x - G and 3y = ax + 5 are parallel. What is the value of a?

**16.** In the triangle figure shown, find the coordinates of the point A and the area of the triangle AOB.




**17.** Do the lines x - 2y + 6 = 0 and -2x + 4y + 6 = 0 intersect?

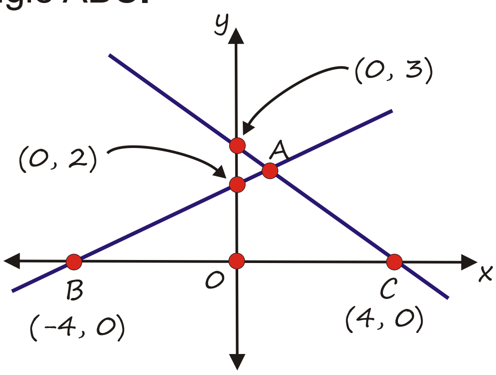
**18.** Find the intersection point of the lines 3x - 4y + 12 = 0 and -6x + 8y - 24 = 0.

**19.** Find the values of k that make the equations 3x - ky + 6 = 0 and (k - 5)x + 2y - 4 = 0

**a.** perpendicular

**b.** parallel.

**20.** Find the area of the triangle ABC.





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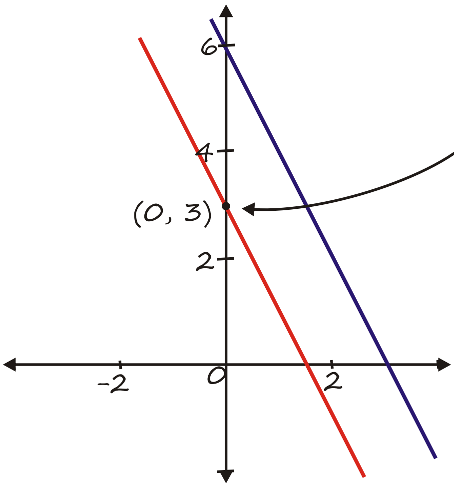
# Another Coordinate Geometry Example

A lot of the questions in this achievement standard will be given in some sort of context however some schools might still give a traditional assessment with questions such as the one below. If you successfully answered this question you would be considered to be at an excellence standard.

e.g. Find the distance between the lines $4x + 2y = 6$ and $4x + 2y = 12$.

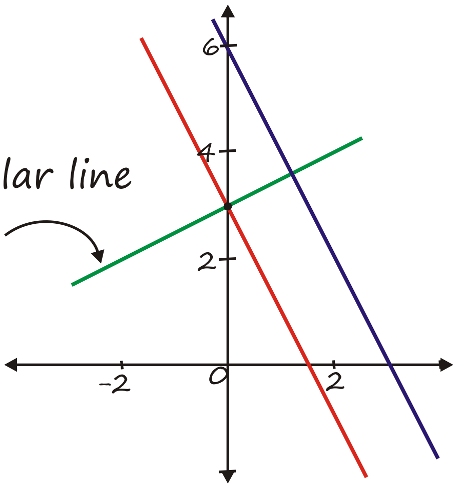
*The steps to solve this problem are set out below.*

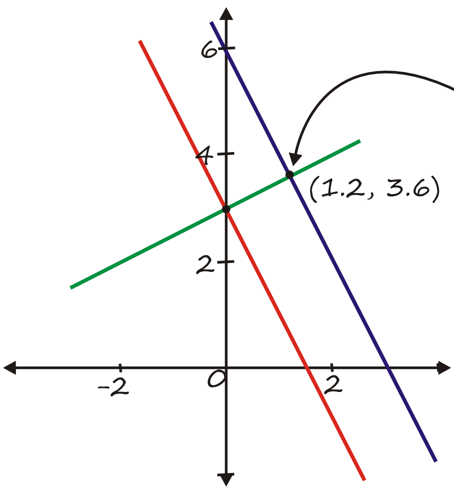
*Equation 1: $4x + 2y = 6$, Equation 2: $4x + 2y = 12$.*



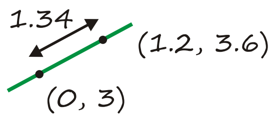

*1. First find a point on one of the lines. Using Equation 1, a point is (0, 3)*

*2. Draw a perpendicular line through that point.*




*3. Find the intersection between the perpendicular and the other parallel line.*




*4. Find the distance between the two points.*

**See all the calculations on the next page.**

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Find the distance between the lines 4x + 2y = 6 and 4x + 2y = 12.

Equation 1: $4x + 2y = 6$

Equation 2: $4x + 2y = 12$

Find a point on one of the lines. e.g on Equation 1: A = (0, 3).

Equation 1 can be rewritten as $2y = -4x + 6$

$$y = -2x + 3$$

The gradient of the line is −2

The gradient of a line perpendicular to 1 will be ½. This is because $-2 \times \frac{1}{2} = -1$ (gradients $m_1m_2 = -1$ for perpendicular)

The equation of the perpendicular line through (0, 3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \tfrac{1}{2}(x - 0)$$

$$y = \tfrac{1}{2}x + 3$$

Find the intersection of $y = \frac{1}{2}x + 3$ and $4x + 2y = 12$

$$4x + 2(\tfrac{1}{2}x + 3) = 12$$

$$4x + x + 6 = 12$$

$$5x + 6 = 12$$

$5x = 6$ therefore $x = 1.2$

Putting the y value into one of the equations $y = \frac{1}{2}(1.2) + 3$

$$y = 3.6$$

Find the distance between (0, 3) and (1.2, 3.6)

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{distance} = \sqrt{(1.2)^2 + (3.6 - 3)^2}$$

distance = 1.34 (units)



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# Exercises

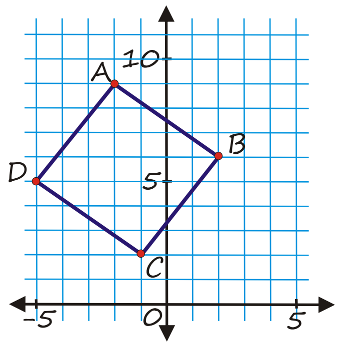
**1.** The shape below is of a square A(-2, 9), B(2, 6), C(-1, 2) and D(-5, 5). The centre of the square is located at the midpoint of the diagonal DB.

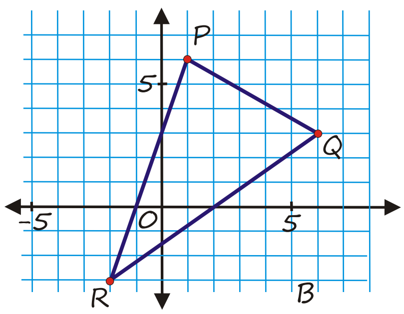
a. Calculate the centre of the square.

b. Calculate the length of DB.

c. Calculate the equation of the line AB.

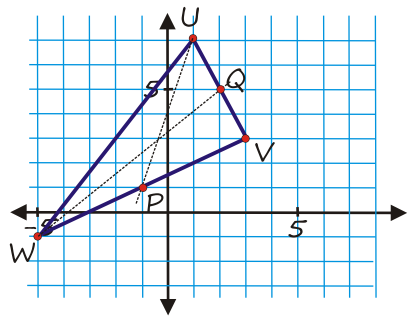
d. Show that line AD is perpendicular to DC.




**2.** The triangle PQR has vertices P(1, 6), Q(6, 3) and R(-2, -3).
An altitude is the line from a vertex that is perpendicular to the opposite side.
Calculate the equation of the altitude from vertex P.

**3.** The diagram shows the triangle U(1, 7), V(3, 3), W(-5, -1).
Point P is the midpoint of WV.
The equation of the median line UP is y = 3x + 4.
The centroid of the triangle UVW is the intersection of the medians UP and WQ.
Find the coordinates of the centroid.




**4.** The lines 2x - y - 5 = 0 and 2x - y + 15 = 0 are parallel.
Calculate the equation of the line that is midway between the two.

**5.** **a.** The island Cirbus is midway between Aark(-4, 1) and Birtrue(2, 4). Calculate the coordinates of Cirbus.

**b.** Give the equation of the line passing through Aark and Birtrue.

**c.** A merchant ship is at point (7, 1) traveling at a perpendicular to the line y = 2x - 4. Is it heading towards Aark or Birtrue?

**d.** A plane leaves the mainland at (0,-4) and flies along the route y = 2x - 4. What is the closest horizontal distance that the plane will get to Aark?

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**6.** The line AB drawn below has points A(-5, 11) and B(5, -3).

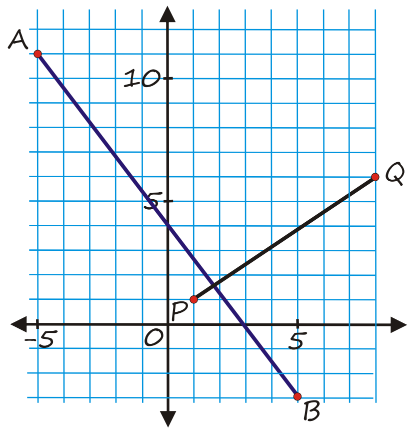
**a.** Find the mid-point of AB.

**b.** Find the length of AB.

**c.** Find the gradient of AB.

**d.** Find the equation of AB.

**e.** A line PQ, perpendicular to AB, is drawn.
Point P is at (1, 1).
What is the equation of the line PQ?




**7.** An ocean beach swim race is being conducted at Mahobe Bay.
The swimmers start at point S(-3, 0).
They swim to point A (-1, 2), then to point B(4, 3), then to point C(3, 6) and finally back to the finish line at O(0, 0).
A small fleet of spectator boats is situated halfway between points A and C. There is a concern by some of the organising committee that this is too close.

**i.** Draw the above situation on some graph paper. Each square represents 10m × 10m.

**ii.** Give all the mathematical information that you can about the race course. For example calculate the distances, angles, gradients, equations and midpoints.

**iii.** Is the swimming leg BC perpendicular to AB? Show your reasoning.

**iv.** One of the organisers is concerned about the course. He points out to the committee that those swimming the final leg OC will clash with those swimming between A and B. Calculate mathematically the point at which the two legs clash and suggest a course change that does not affect the overall race distance and where none of the legs clash.

**v.** Another concern is that spectator boats may be too close to the swimmers. Therefore the organising committee make a rule that all spectator boats must be outside an imaginary line that runs parallel to the swim leg AC. This line goes through point (1, 5). As part of your calculations comment on whether this line will be a safe distance from the swimmers. If it is not okay, suggest where it could be so that the boats are a safe distance from the swimmers.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**8.** A yachting match race is between two boats around one of 2 courses set on the water. The race officials usually decide on the course the morning of the race dependant on wind conditions. One particular race has been set out on a grid with each square representing 1 nautical miles.
Course 1 is 6 legs between points A(-1, 1) and B(8, 7)
Course 2 is 6 legs between points P(5, 2) and Q(-1, 10)

- **a.** Draw the 2 courses AB and PQ on some graph paper.
- **b.** Calculate the equation of the line AB.
- **c.** Calculate the equation of the line PQ.
- **d.** Is the second course (PQ) perpendicular to the first course (AB)?
- **e.** Calculate the total distance for a yacht race around the course AB. (Remember there are 6 legs to each race.)
- **f.** What is the difference in distance between each of the courses AB and PQ?

Officials have placed markers at the half way point on each of the courses.

- **g.** Find the midpoint of each of the courses AB and PQ.
- **h.** Find the intersection point of the 2 courses AB and PQ.

**9.** An orienteering course is run around a park. If plotted on a grid, the runners start at A(-12, 12) and then run to point B(6, 3). They turn right and run to point C(2, -5) and then to point D(-6, -1). Finally they run back to the finish line at point A(-12, 12).

- **a.** Plot the course on some graph paper. If each unit square is 10 000 metres$^2$ then find the total distance around the course.
- **b. i.** The women orienteers run part of the men's course. They run along AB but only run half way along BC. This means that they turn at the mid point of BC. At what point on the grid do they turn?
  - **ii.** From this point they then run to the mid-point of DA. What is the midpoint of DA?
- **c.** Willis is wondering whether legs AB and CD are parallel. Find the equations of both legs and prove whether they are parallel or not.
- **d.** If lines BC and AD were extended they would intersect. Find the equations of both the lines CB and AD and find the point at which they would intersect.

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**10.** Diagram A shows a triangle FGH with all 3 vertices on the circumference of a circle.

**a.** Is the triangle an isosceles**,** equilateral or scalene?

**b.** Use coordinate geometry to find the circumcentre and the radius of the circle.

**Hint** The circumcentre of a triangle is where all the perpendicular bisectors meet.

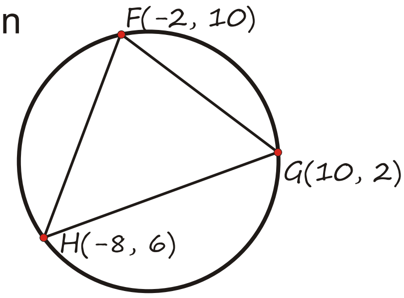


Diagram A

Diagram B shows the same triangle however this time it shows the centroid. The centroid is the center of the triangle and could be used as the balancing point on the tip of a pencil.

**c.** Find the centroid of the triangle.

**Hint** The centroid of a triangle is the point of intersection of its medians (the lines joining each vertex with the midpoint of the opposite side).

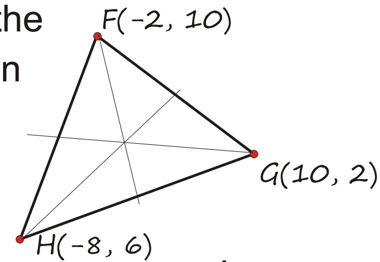


Diagram B

**11.** Three straight line equations are given below.

$$2x - 6y - 4 = 0$$
$$2x + 2y - 8 = 0$$
and $$7x - 3y + 22 = 0$$

**a.** Show that the lines intersect at the points A(-1, 5), B(3.5, 0.5) and C(-4, -2)

**b.** The orthocentre of a triangle is the point where the three altitudes of that triangle intersect. Calculate the orthocentre of triangle ABC.

**12.** A triangle is formed by joining the points A(0, 0), B(6, 13) and C(10, 8).

**a.** Find the equation of each line that forms the triangle.

**b.** Find the midpoint of each side of the triangle. Call these points x, y, z.

According to mathematical theory, joining the midpoints will form a second smaller triangle that has an area ¼ the size of the original.

**c.** Find the area of ABC and the area of xyz.
Does your calculation indicate that this theory could be correct?

**13.** Point A is (-5, 5), point B is (7, -1) and point C is (a, 2a).
Triangle ABC is isosceles where AC = BC.
The area of ABC is 30 units².
Find the coordinates of C.



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# AS91256 Coordinate Geometry
# What is Involved?

**Achievement**

In most instances a problem will be given within some sort of context. However some schools might just give a traditional test that involves a number of questions that are progressively more challenging. To gain an 'achievement' grade you will need to demonstrate that you can find a midpoint, gradient and equation of a line. You may also have to show that you can recognise a parallel or perpendicular line by comparing the different gradients and to do this you may have to understand terms such as "perpendicular bisector".

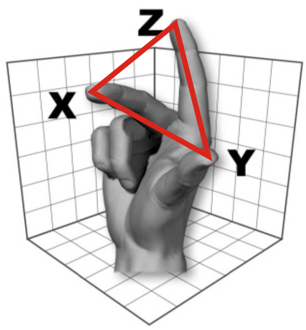
**Merit**

To gain a 'merit' grade requires you to carry out a sequence of steps that link together the different concepts above. For example you may be given 2 pairs of points joined by lines. For merit, you might need to find the intersection of the two lines. You would do this by finding the gradients of the lines, the equations of the lines and then solve the equations simultaneously to find the point of intersection. At each stage you need to be able to explain what you are calculating and why you are doing it that particular way.

**Excellence**

To gain an 'excellence' grade requires "extended abstract thinking". This takes the merit standard a little further. You should be able to explain the logic behind any answers you produce and be able to justify them with appropriate mathematical statements. An example of this could be a triangle in a circle. If you can show that one angle is a right angle then the longest side must be a hypotenuse and the diameter of the circle. This is because angles in a semi-circle equal 90°. Quite often excellence questions will involve variables instead of numbers so you need to be able to manipulate algebraic equations.


*All calculations in this booklet were performed using a*
**Mahobe DS-742ET**

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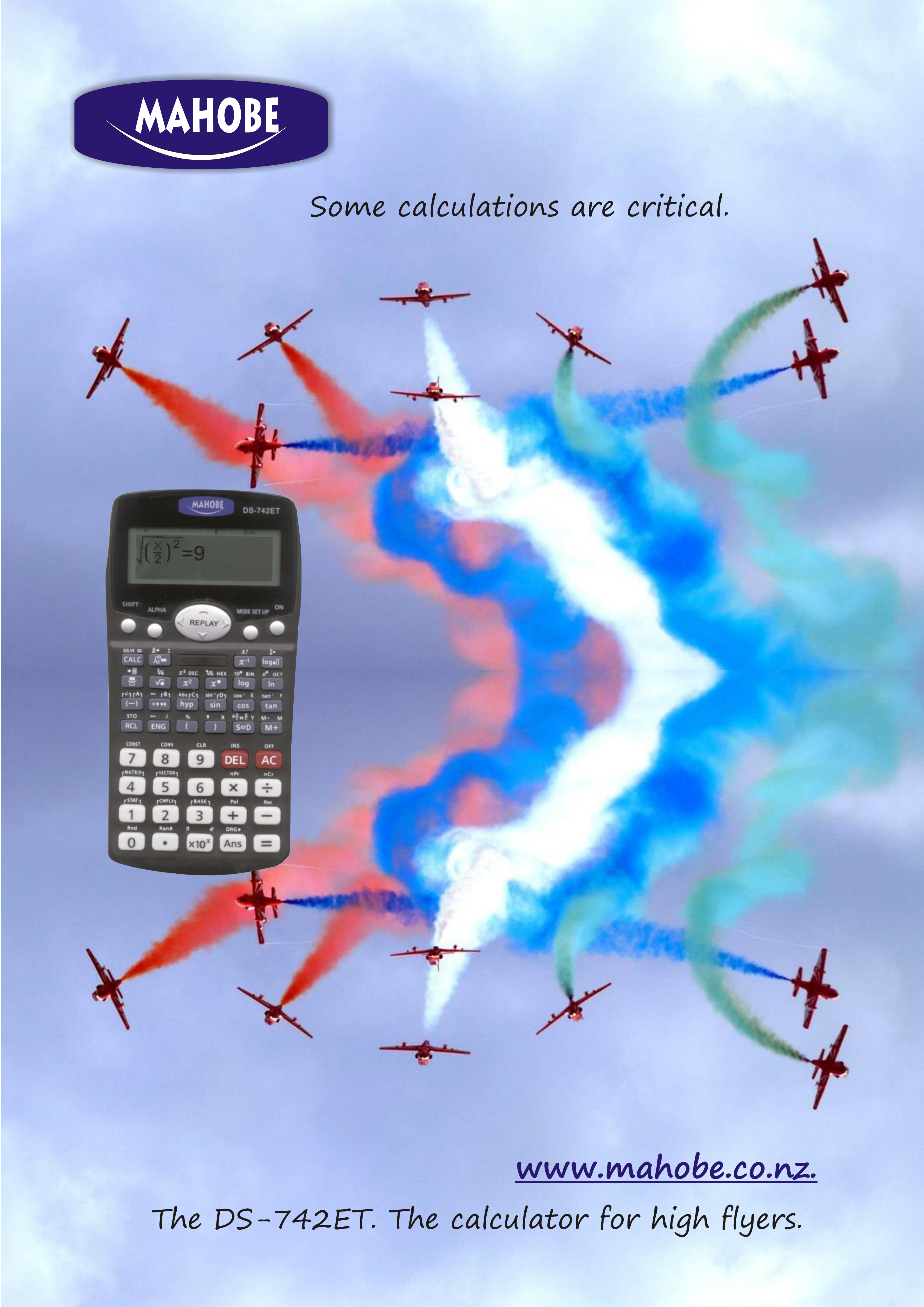

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Some calculations are critical.
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The DS-742ET. The calculator for high flyers.

# The Answers

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**1.a.** midpoint $x(-6+8)\div 2$, $y(7+0)\div 2$

midpoint $(x, y) = (1, 3.5)$

**1b.** length $= \sqrt{(-6-8)^2 + (7)^2}$

$= 15.65$ units

**1c.** The gradient between the points is

$7 \div (-6 - 8) = -\tfrac{1}{2}$

The equation $y - y1 = m(x - x1)$

Using $(8, 0)$ $\quad y = -\tfrac{1}{2}(x - 8)$

$y = -\tfrac{1}{2}x + 4$

**1d.** Parallel line through $(1, 1)$ gradient $-\tfrac{1}{2}$.

$y - 1 = -\tfrac{1}{2}(x - 1)$

$y = -\tfrac{1}{2}x + \tfrac{1}{2} + 1$

$y = -1.5x + 1.5$ $\quad$ or $\quad y = \frac{-x}{2} + \frac{3}{2}$

**2.** The gradient of $y = 2x - 5$ is 2.

Perpendicular gradient $= -\tfrac{1}{2}$ as $m_1m_2 = -1$.

$y + 1 = -\tfrac{1}{2}(x - 4)$

$y + 1 = -\tfrac{1}{2}x + 2$

$y = -\tfrac{1}{2}x + 1$

**3.** The gradient of the parallel line = 4

$y + 2 = 4(x + 2)$

$y + 2 = 4x + 8$

$y = 4x + 6$

**4.** $y = x - 10$, $2x + 3y = 55$

Combining the two equations

$2x + 3(x - 10) = 55$

$2x + 3x - 30 = 55$

$5x - 30 = 55$

$5x = 85$, $\quad x = 17$

Putting $x = 17$ into one of the equations

$y = x - 10$

$y = 17 - 10$

$y = 7$, $\quad$ therefore the POI is $(17, 7)$.

**5.** The gradient between A and C $= \frac{7 + 20}{2 - 20}$

$= \frac{-3}{2}$ or $-1.5$

Equation through $(2, 7)$ using $m = -1.5$

$y - 7 = -1.5(x - 2)$

$y = -1.5x + 10.$

Therefore using $B(6, k)$ and the equation.

$y = -1.5(6) + 10$

$y = 1,$

i.e. $\quad k = 1$

**6.** $JK = \frac{-1 - 8}{-3 - 0} = 3$

$JL = \frac{-1 - 2}{-3 - 3} = \frac{1}{2}$

$KL = \frac{8 - 2}{0 - 3} = -2$

The gradients $JL \times KL = -1$.

This indicates that there is a pair of perpendicular lines and therefore a right angled triangle.

**7.** Put both in the form $y = mx + c$

$8x + 5y = 10$

$5y = 10 - 8x$

$y = 2 - \frac{8}{5}x$ $\quad$ i.e gradient $= \frac{-8}{5}$

$10x + ky = 11$

$ky = 11 - 10x$

$y = \frac{11}{k} - \frac{10}{k}x$ $\quad$ i.e. gradient $= \frac{-10}{k}$

For perpendicular lines $m_1m_2 = -1$

$\frac{-8}{5} \times \frac{-10}{k} = -1$ (or $\frac{80}{-80}$ )

$k = -16$

Therefore the equation is $10x - 16y = 11$

**8.** The gradient of RS is $\frac{5 + 2}{-5 - 9} = \frac{7}{-14}$

i.e. the gradient $= \frac{-1}{2}$

Equation $y - 5 = -\tfrac{1}{2}(x + 5)$

$y = -0.5x - 2.5 + 5$

$y = -0.5x + 2.5$

Therefore using $V(k, -10)$ where $x = k$

$-10 = -0.5k + 2.5$

$-12.5 = -0.5k$

$k = 25$

**9.a.** Use $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

OA = 25, OB = 25 and AB = 30 (units)

Therefore the side lengths are 25, 25, 30 indicating an isosceles triangle.

**9.b.** Gradient of AB $= \frac{24}{7 - 25} = \frac{-4}{3}$

Gradient of perpendicular line $= \frac{3}{4}$

Equation of AB using point $B(25, 0)$

$y = \frac{-4}{3}x + \frac{100}{3}$

Equation of perpendicular $y = \frac{3}{4}x$

**9.c.** The point where they intersect = $(16, 12)$

Use the midpoint formula to show that $(16, 12)$ is the midpoint of AB.

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**10.** $x + y + 2 = 0$

$\underline{x - y - 8 = 0}$

$2x \quad - 6 = 0$ (adding both together)

$x = 3$

Now putting 3 into the first equation

$3 + y + 2 = 0, y = -5$

This means that the intersection point is $(3, -5)$

rearranging both equations to $y = mx + c$

$y = -x - 2$ gradient $= -1$

$y = x - 8$ gradient $= 1$

As $\tan 45° = 1$ it means that the lines are each at 45° to the horizontal. Therefore the line $y = -5$ must be the horizontal bisector.

**11.** The equation between (3, 1) and (0, 0) must have a gradient of 1/3. This means that the tangent must have a gradient of −3. Therefore the equation of the line at the point (3, 1) with a gradient −3

$y - 1 = -3(x - 3)$

$y = -3x + 10$

**12.** The line $y = 3x - 5$ has a gradient of 3. The gradient of the perpendicular $= -1/3$

Therefore the equation of the line through the point (5, −3) with gradient −1/3

$y + 3 = \frac{-1}{3}(x - 5)$

$y = \frac{-x}{3} - \frac{4}{3}$

Finding the intersection point of

$y = 3x - 5$ and $y = \frac{-x}{3} - \frac{4}{3}$

$3y = 9x - 15$ and $3y = -x - 4$

$9x - 15 = -x - 4$

$10x = 11, x = 1.1$

If $x = 1.1$, using $y = 3x - 5$, $y = -1.7$

distance between (5, −3) and (1.1, −1.7)

$\sqrt{(5 - 1.1)^2 + (-3 + 1.7)^2}$

$= 4.1$ units

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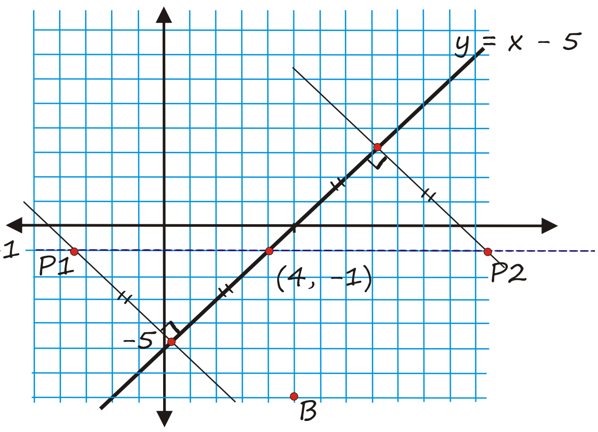
**13.**




The diagram above shows the situation. There are two right angled triangles which are also isosceles. The line $y = -1$ intersects $y = x - 5$ at the point (4, −1).

Because the distance is 6 units the hypotenuse of each triangle

$= \sqrt{6^2 + 6^2}$

$= 8.5$ (1 dp)

Therefore the points P1 and P2 are

$4 \pm 8.5$ (this is the distance from the intersection point $x = 4$)

Therefore the points are (12.5, −1) and (−4.5, −1)

**14.** Both equations have a gradient of 1.5 (3/2) therefore they must be parallel.

Take a point on one of the lines e.g.

$-3x + 2y + 12 = 0$ has a point (4, 0)

Gradient of the perpendicular line is $\frac{-2}{3}$

Using $y - y_1 = m(x - x_1)$

$y = \frac{-2}{3}(x - 4)$

$3y = -2x + 8$

Find the intersection between $2x + 3y - 8$ and $3x - 2y + 9 = 0$ gives the points

$x = -0.846$ and $y = 3.23$ (calculator)

The distance between (4, 0) and (−0.846 and 3.23)

$\sqrt{(4 + 0.846)^2 + (-3.23)^2} \quad = 5.82$ units



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**Page 12**

**15.** First look at the side lengths

AB = 5, DC = 5, AD = 5, BC = 5

Slope of AD = $\frac{3}{4}$ Slope of DC = $\frac{-4}{3}$

Therefore ABCD is a square. All sides are the same length and the opposite angles are right angles.

**16.** The midpoint of AC = (0, 5) and the midpoint of BD = (0, 5). As the diagonals bisect each other the shape must be a parallelogram.

**17a.** distance $= \sqrt{(16-4)^2 + (10-6)^2}$

$= 12.65$ m

**17b.** $x = (16 + 4) \div 2$ and $y = (6 + 10) \div 2$

$= (10, 8)$

**17c.** gradient = $\frac{1}{3}$

$y - y1 = m(x - x1)$, $y - 6 = \frac{1}{3}(x - 4)$

$y = \frac{1}{3}x - \frac{4}{3} + 6$

$3y = x - 4 + 18$

$3y = x + 14$

$3y - x - 14 = 0$

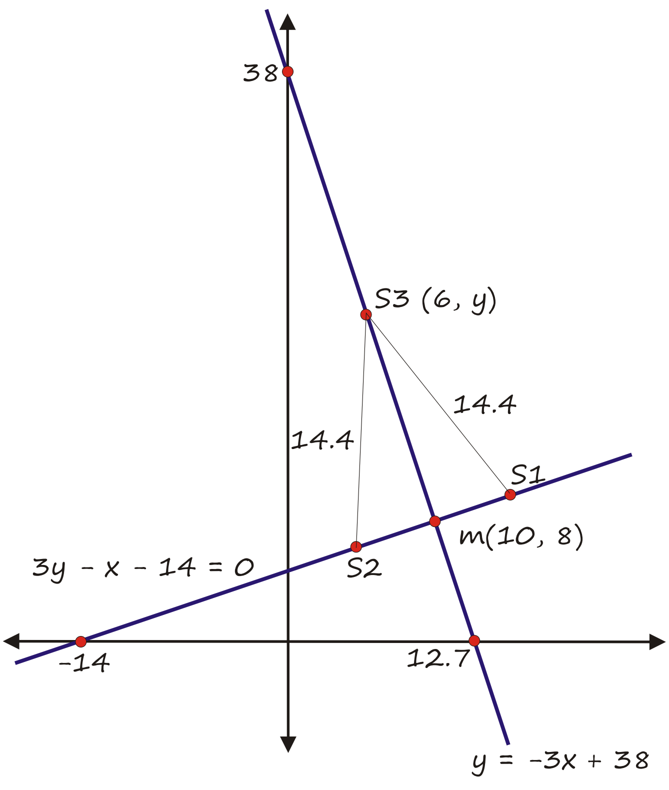
**17d.** gradient of perpendicular = $-3$

$y - 8 = -3(x - 10)$

$y - 8 = -3x + 30$

$y = -3x + 38$

**17e.**




**17e.** The distance between S1 and S3

$14.14 = \sqrt{(4 - 6)^2 + (6 - y)^2}$

$199.94 = 4 + 36 - 12y + y^2$

$0 = -159.94 - 12y + y^2$

Using the calculator equation solver

$y = 20$ or $-8$

Therefore the coordinates of S3 are (6, 20)

Note that the point (6, -8) may be 14.14m away from S1 but it is not on the line $y = -3x + 38$


The New Zealand Centre of Mathematics recommends the DS-742ET calculator. It shows equations the same way they are written in textbooks and it has a built in equation solver.

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**Page 15**

1. The centre will be the midpoint of the diameter.
$x = (-15 + 7) \div 2, x = -4$
$y = (2 + 12) \div 2, y = 7$
Therefore centre point is $(-4, 7)$.

2. In this example use the same equations as in number 1.
$(x + 2) \div 2 = 8$, i.e. $x + 2 = 16$
$(y - 3) \div 2 = 1$, i.e $y - 3 = 2$
$x = 14, y = 5$
Therefore point G is $(14, 5)$

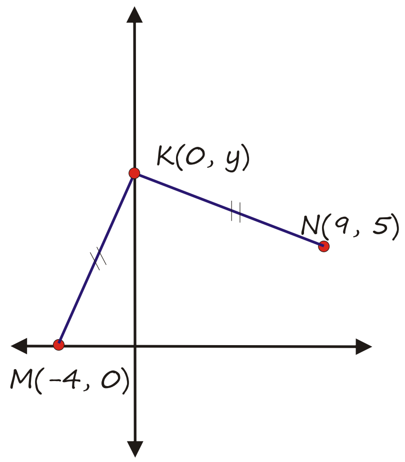
3. If the hole is at $(0, 0)$ then the first shot would land at $(8, -25)$. The second shot landed at $(-1, 3)$.
The distance is $\sqrt{(8+1)^2 + (-25 - 3)^2}$
$= 29.41$ metres

4. Using the distance formula
$\sqrt{(7 - x)^2 + (5 + 3)^2} = 10$
$(7 - x)^2 + (8)^2 = 100$
$49 - 14x + x^2 + 64 = 100$
$x^2 - 14x + 13 = 0$
$(x - 13)(x - 1) = 0$
$x = 13$ or $x = 1$
There are two possible answers
$(13, -3)$ or $(1, -3)$

5. To find the bisection (midpoint) of RS
$x = (-4 + 10) \div 2, x = 3$
$y = (9 + 7) \div 2, y = 8$
Midpoint $= (3, 8)$
The gradient of the chord RS
$= (9 - 7) \div (-4 - 10)$
$= -1/7$
Therefore the gradient of the perpendicular line is 7.
The equation through $(3, 8)$, gradient $= 7$
$y - 8 = 7(x - 3)$
$y - 8 = 7x - 21$
$y = 7x - 13$
Checking to see if the equation passes through point $O(2, 1)$ $y = 7x - 13$
$1 = 14 - 13$
Therefore yes it passes through the centre.

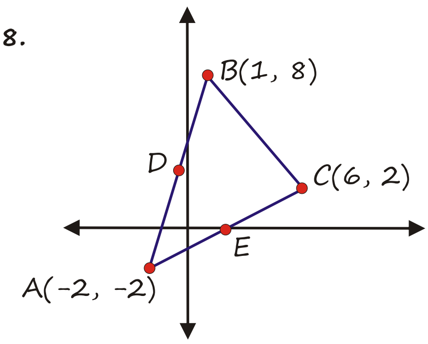
6. Use the distance formula
$10 = \sqrt{(2 + 6)^2 + (a - 1)^2}$
$100 = 64 + a^2 - 2a + 1$
$a^2 - 2a - 35 = 0$
$(a - 7)(a + 5) = 0$
$a = 7$ or $a = -5$
Therefore there are two possible values for $a$, 7 or $-5$

7. Here is a diagram of the situation.




$MK = NK$
therefore use the distance formula
$\sqrt{(-4)^2 + (y)^2} = \sqrt{9^2 + (y - 5)}$
$(-4)^2 + (y)^2 = 9^2 + (y - 5)$
$16 + y^2 = 81 + y^2 - 10y + 25$
$10y = 90$
$y = 9$
Therefore the point K is $(0, 9)$.

8.




Midpoint of AB is x: $(-2 + 1) \div 2 = -½$
y: $(-2 + 8) \div 2 = 3$
Midpoint of AC is x: $(-2 + 6) \div 2 = 2$
y: $(-2 + 2) \div 2 = 0$
Midpoints are $D(-½, 3)$ and $E(2, 0)$.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**Page 15 (cont)**

**8 (cont)**

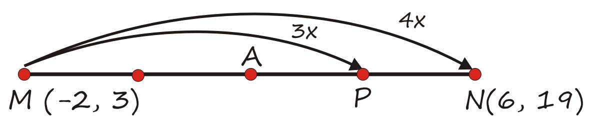
$$ED = \sqrt{(2 + ½)^2 + (0 - 3)^2} = \frac{\sqrt{61}}{2}$$

$$BC = \sqrt{(6 - 1)^2 + (2 - 8)^2} = \sqrt{61}$$

Therefore ED = ½BC

Note how it is sometimes easier with surds (numbers with square roots in them) to calculate accurate answers. For example the square root of 61 is 7.8 (1 dp). If we square 7.8 we get 60.84. This does round to 61 but if wanting an accurate figure the figure $\sqrt{61}$ is better.

**9.** Divide the line into 4 equal parts.




A = half the distance between M and N

$x = (-2 + 6) \div 2$

$y = (3 + 19) \div 2$

Point A = (2, 11)

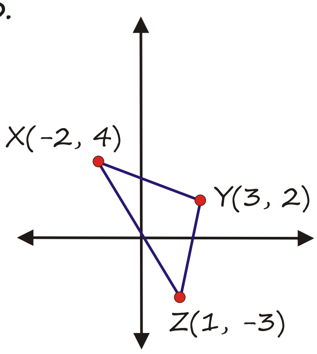
P is half the distance between A and N

$x = (2 + 6) \div 2$

$y = (11 + 19) \div 2$

Point P = (4, 15)

**10.**




Upon drawing a sketch it appears that YZ and YX could be perpendicular. If so then this will save a lot of calculations.

Gradient of YZ: $(-3-2) \div (1-3) = \frac{5}{2}$

Gradient of YX: $(2 - 4) \div (3+2) = -\frac{2}{5}$

Multiplying the two gradients gives −1 therefore they must be perpendicular.

Area of a triangle = ½ base × height

Base length = YZ

Height length = YX

$$YZ: \sqrt{(3 - 1)^2 + (2 + 3)^2} = \sqrt{29}$$

$$YX = \sqrt{(3 + 2)^2 + (2 - 4)^2} = \sqrt{29}$$

$$\begin{aligned} \text{area} &= ½YZ \times YX \\ &= ½ \times \sqrt{29} \times \sqrt{29} \\ &= 14.5 \text{ (units)}^2 \end{aligned}$$

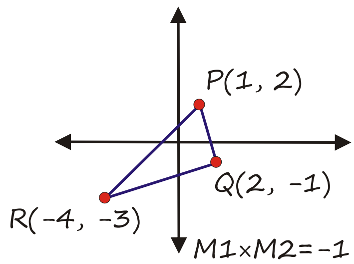
**11.** To be co-linear the gradient of FG should be the same as GH

Gradient FG: $(1+2) \div (4-5) = -3$

Gradient GH: $(-2+5) \div (5-6) = -3$

Gradients are equal therefore the points must be co-linear.

**12.** The sketch below shows a possible right angle between PQ and QR.



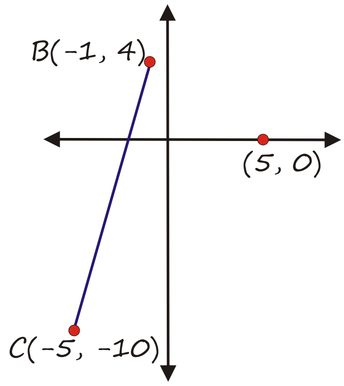

$M1 \times M2 = -1$ for perpendicular

Gradient PQ: $(2+1) \div (1-2) = -3$

Gradient QR: $(-1+3) \div (2+4) = \frac{1}{3}$

**Page 16**

**13.**




Gradient of BC

$(4 + 10) \div (-1 + 5) = \frac{7}{2}$

Using the gradient and the point (5, 0)

$$y - y_1 = m(x - x_1)$$

$$y = \frac{7}{2}(x - 5)$$

$$2y = 7(x - 5)$$

$$2y - 7x + 35 = 0$$

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**Page 16 (cont)**

**14.** In the diagram AB = BC
Therefore let the point B have the co-ordinates $(-a, a)$
The point $B(-a, a)$ is also on the line BE.
Therefore substitute the point into the equation.

$$y = \sqrt{3}x + 3\sqrt{3} + 3$$
$$a = \sqrt{3}(-a) + 3\sqrt{3} + 3$$
$$a + \sqrt{3}a = 3(\sqrt{3} + 1)$$
$$a(1 + \sqrt{3}) = 3(\sqrt{3} + 1)$$
$$a = 3$$

Therefore the square OABC has an area of $(3^2) = 9$ square units.

**15.** If $y = 2x - G$ then $3y = 6x - 3G$
We don't need to worry about the 3G part of the equation as this is just the intercept. The important part for this equation is the gradient (6x). Therefore the value of $a = 6$.

**16.** First find the equation of AB
Gradient = $-\frac{1}{2}$ , y intercept = 2
Equation is $y = -\frac{1}{2}x + 2$
or $x + 2y - 4 = 0$
Equation of AO is $y = -x$
Point A is the intersection of $y = -x$ and $x + 2y - 4 = 0$
$x + 2(-x) - 4 = 0$
$x - 2x - 4 = 0$, $x = -4$ and $y = 4$,
therefore point A = $(-4, 4)$
The area of the triangle with OB = 4 and altitude = 4
$\frac{1}{2} \times 4 \times 4 = 8$ square units

**17.** $2x - 4y + 12 = 0$ (equation 1 × 2)
$-2x + 4y + 6 = 0$
$18 = 0$
Adding both together shows that the lines are parallel and therefore do not intersect. If you multiplied equation 2 by −1 you would get $2x - 4y - 12 = 0$. The x and the y values are the same. The only point of difference is the intercept value. The gradients are the same so the lines will not intersect.

**18.** $3x - 4y + 12 = 0$
$\underline{-6x + 8y - 24 = 0}$
$6x - 8y + 24 = 0$
$\underline{-6x + 8y - 24 = 0}$
$0 = 0$
They are the same line. Therefore there are an infinite number of solutions (i.e. they are coincident).

**19.** $3x - ky + 6 = 0$
$(k - 5)x + 2y - 4 = 0$
Find the gradients
a. $ky = 3x + 6$ therefore gradient = $\frac{3}{k}$
b. $2y = -(k - 5)x + 4$
$2y = (5 - k)x + 4$, gradient = $\frac{5 - k}{2}$
For perpendicular $a \times b = -1$

$$\frac{3}{k} \times \frac{5 - k}{2} = -1$$
$$\frac{15 - 3k}{2k} = -1$$
$$15 - 3k = -2k$$
$$k = 15$$

For parallel lines $a = b$

$$\frac{3}{k} = \frac{5 - k}{2}$$
$$6 = 5k - k^2$$
$$k^2 - 5k + 6 = 0$$

$k = 3$ or $k = 2$ (two possible answers)

**20.** There are two equations.
AB = $\frac{1}{2}x + 2$
AC = $-\frac{3}{4}x + 3$
Using the DS-742ET calculator the intersection point A is (0.8, 2.4)
Therefore the height of the triangle is 2.4.
The base length is 8
Area is $\frac{1}{2} \times 8 \times 2.4 = 9.6$ square units.



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**Page 19**

**1.a.** There are two ways of finding the centre of the square.

i. Find the equations of the lines AC and DB and then solve simultaneously.

ii. Find the equations through the midpoints of AB and DC and the equations of the midpoints of AD and BC and then solve simultaneously.

Of the two options the first takes the least amount of calculations.

Equation of AC coordinates (−2, 9), (−1, 2)

Gradient AC is $\frac{9-2}{-2--1} = -7$

Equation AC

$y - y_1 = m(x - x_1)$

$y - 9 = -7(x - -2)$

$y - 9 = -7x - 14$

$y = -7x - 5$

Gradient of DB coordinates (−5, 5), (2, 6)

Gradient DB is $\frac{5-6}{-5-2} = \frac{1}{7}$

Equation DB

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{7}(x + 5)$

$y - 5 = \frac{1}{7}x + \frac{5}{7}$

$y = \frac{1}{7}x + 5\frac{5}{7}$

or $7y = x + 40$

Solving the equations simultaneously

$y = -7x - 5$

$7y = x + 40$

$7(-7x - 5) = x + 40$

$-49x - 35 = x + 40$

$-50x = 75$

$x = -1.5$

Using $y = -7x - 5$ and $x = 1.5$

$y = 5.5$

**1b.** Length of BD coordinates (2, 6)(−5, 5)

$= \sqrt{(2+5)^2 + (6-5)^2}$

$= \sqrt{50}$

**1c.** AB (−2, 9)(2, 6)

Gradient: $\frac{9-6}{-2-2} = \frac{-3}{4}$

Equation $y - y_1 = m(x - x_1)$

$y - 9 = \frac{-3}{4}(x + 2)$

$y - 9 = \frac{-3}{4}x - \frac{6}{4}$

$y = \frac{-3}{4}x + 7\frac{1}{2}$

$4y = -3x + 30$

**1d.** AD (−2, 9)(−5, 5)

Gradient: $\frac{9-5}{-2+5} = \frac{4}{3}$

DC (−5, 5)(−1, 2)

Gradient: $\frac{5-2}{-5+1} = \frac{-3}{4}$

$\frac{4}{3} \times \frac{-3}{4} = -1$

Therefore the lines are perpendicular.

**2.** Find the gradient of RQ (−2, −3)(6, 3)

$\frac{-3-3}{-2-6} = \frac{3}{4}$

This means the gradient of any line perpendicular to RQ would be $\frac{-4}{3}$

The equation of the altitude line through P(1, 6)

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{-4}{3}(x - 1)$

$y - 6 = \frac{-4}{3}x + \frac{4}{3}$

$y = \frac{-4}{3}x + \frac{22}{3}$

or $3y + 4x = 22$

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**3.** Midpoint UV (1, 7)(3, 3)

$\frac{1+3}{2} \qquad \frac{7+3}{2}$

= (2, 5)

Gradient of WQ (−5, −1) (2, 5)

$\frac{-1-5}{-5-2} = \frac{6}{7}$

Equation WQ using $m = \frac{6}{7}$ and (−5, −1)

$y - y_1 = m(x - x_1)$

$y + 1 = \frac{6}{7}(x + 5)$

$y + 1 = \frac{6}{7}x + \frac{30}{7}$

$y = \frac{6}{7}x + \frac{23}{7}$

$7y = 6x + 23$

Intersection

$y = 3x + 4$

and $7y = 6x + 23$

$7(3x + 4) = 6x + 23$

$21x + 28 = 6x + 23$

$15x = -5$

$x = -\frac{1}{3}$

using $y = 3x + 4$ and $x = -\frac{1}{3}$

$y = 3$

4. $2x - y - 5 = 0$

$2x - y + 15 = 0$

Rearrange the equations to get the gradient ($y = mx + c$)

$y = 2x - 5$ and $y = 2x + 15$

Therefore the gradient of both is 2.

Take a point on $y = 2x - 5$

e.g. $x = 5$, $y = 10 - 5$

therefore $y = 5$ and the point (5, 5)

The gradient of the perpendicular line to both equations is $-\frac{1}{2}$ ($m_1m_2 = -1$)

Therefore the equation through (5, 5)

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 5)$

$2y - 10 = -x + 5$

$2y = -x + 15$

4 (continued).

Using $2y = -x + 15$

and $2x - y + 15 = 0$

or $4x - 2y + 30 = 0$

Then $4x - (-x + 15) + 30 = 0$

$5x + 15 = 0$

$x = -3$

Substituting $x = -3$ back into one of the equations.

$2y = 3 + 15$

$y = 9$

The 2 points are (−3, 9) and (5, 5)

The midpoint between is:

$\frac{-3+5}{2} \qquad \frac{9+5}{2}$

= (1, 7)

Using gradient 2 and the point (1, 7)

$y - y_1 = m(x - x_1)$

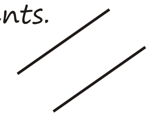
$y - 7 = 2(x - 1)$

$y - 7 = 2x - 2$

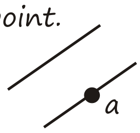
$y = 2x + 5$

Below are the steps that were followed

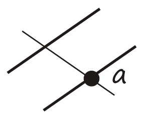
1. Find the gradients.



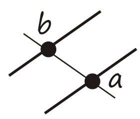
2. Choose a point.



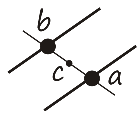

3.Find perpendicular line through point.



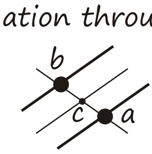

4. Find second point.




5. Find the midpoint between a and b.




6. Find the equation through the midpoint.





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**Page 19**

**5a.** A(–4, 1) B(2, 4)

Cirbus $= \dfrac{-4+2}{2}, \dfrac{1+4}{2}$

$= (-1, 2.5)$

**5b.** Find the gradient: $\dfrac{1-4}{-4-2} = \dfrac{1}{2}$

Use the gradient through B(2, 4)

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{2}(x - 2)$

$2y - 8 = x - 2$

$2y = x + 6$

or use the gradient through A(–4, 1)

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x + 4)$

$2y - 2 = x + 4$

$2y = x + 6$

**5c.** The gradient of $y = 2x - 4$ is 2.

The gradient of the perpendicular is $\frac{-1}{2}$

The equation of the line through (7, 1) with gradient $\frac{-1}{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{-1}{2}(x - 7)$

$2y - 2 = -x + 7$

$2y = -x + 9$

Using graphing software such as DESMOS, the line appears to be heading towards Birtrue.

You could answer this one using a simple sketch. At point (7, 1) can only be perpendicular to B.

**5d.** The closest horizontal distance from (–4, 1) will be the line perpendicular to $y = 2x - 4$ that goes through (–4, 1).

Gradient of the perpendicular = $\frac{-1}{2}$

Equation through (–4, 1)

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{-1}{2}(x + 4)$

$2y - 2 = -x - 4$

$2y = -x - 2$

**5d cont**

The intersection point between $2y = -x - 2$ and $y = 2x - 4$

$2(2x - 4) = -x - 2$

$4x - 8 = -x - 2$

$5x = 6$

$x = \frac{6}{5}$

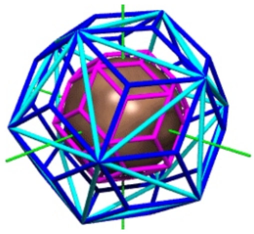
Using $y = 2x - 4$ and $x = \frac{6}{5}$ the intersection point is (1.2, –1.6).

Distance between (–4, 1) and (1.2, –1.6)

$= \sqrt{(-4 - 1.2)^2 + (1 + 1.6)^2}$

$= 5.8$


The New Zealand Centre of Mathematics recommends the DS-742ET calculator. It shows equations the same way they are written in textbooks and it has a built in equation solver.



Calculating coordinates becomes more fun when you have a good calculator.

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**Page 20**

**6.a.** Midpoints

$x = (-5 + 5) \div 2 = 0$

$y = [11 + (-3)] \div 2 = 4$

Therefore the midpoint is (0, 4)

**b.** Length of AB

Length $= \sqrt{[5 - (-5)]^2 + [11 - (-3)^2]}$

Length $= \sqrt{100 + 196}$

Length = 17.20 units

**c.** Gradient of AB = $(-3 - 11) \div (5 - (-5))$

Gradient = $-1.4$ ($\frac{-7}{5}$)

**d.** Using the point (5, −3) and the gradient −1.4

$y - y_1 = m(x - x_1)$

$y + 3 = -1.4(x - 5)$

$y = -1.4x + 7 - 3$

$y = -1.4x + 4$

**e.** The gradient of AB = $\frac{-7}{5}$

Gradient of perpendicular = $\frac{5}{7}$ (or 0.71)

This is because $m_1m_2 = -1$

Equation of the line through (1, 1)

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{5}{7}(x - 1)$

$y = \frac{5}{7}x - \frac{5}{7} + 1$

$y = \frac{5}{7}x + \frac{2}{7}$

**7.** To get Achievement you need to correctly calculate at least one distance, midpoint, gradient and equation. You should also recognise that each unit is 100 m.

Distances

SA = 2.82, AB = 5.10, BC = 3.16, CO = 6.71, Total Distance = 17.79 units (1.779 km)

Angles and Gradients:

SA = 1, AB = 1/5, BC = −3, CF = 2

Equations:

SA $y = x + 3$, AB: $y = x/5 + 11/5$

BC: $y = 15 - 3x$, CO: $y = 2x$

Midpoint of AC (spectators) = (1, 4)

You may calculate other midpoints as well.

<u>Merit Standard Answers</u>

BC and AB are not perpendicular as the gradients are −3 and 1/5 ($m_1m_2 <> -1$)

Equation of AB is $y = 1/5x + 11/5$

Equation of FC is $y = 2x$

Intersection point is (1.22, 2.44)

Distance of CF must equal 6.71 units. Therefore by using vectors the course can be changed to (6, 0) forming an isosceles triangle with the original course and the beach. You may also choose to move the start or points A, B or C. However you must justify the new course and it should still be around the 1.7 to 1.8 kms in total distance.

<u>Excellence Standard Answers</u>

Equation of AC $y = x + 3$

Equation spectator line $y = x + 4$

Gradient perpendicular line = −1

Line through (1, 5), $y = -x + 6$

Intersection of $y = -x + 6$ & $y = x + 3$ is (1.5, 4.5)

The shortest distance between (1, 5) and (1.5, 4.5) is 0.707

Therefore the shortest distance is 70m from the swimmers.



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**Page 20 (continued)**

7. Excellence Standard Answers

Reason whether 70 m is a satisfactory distance apart given that the spectators want to see the race. There will also be different boats and boat sizes, winds, tides currents, wave heights etc. If buoys are used to mark each point then they might move during the swim due to the tide or currents. Answers may also take into account the extra distance used by swimmers when they swim around each point. They never swim in a direct line and they never round the buoys exactly as you might on a piece of paper.

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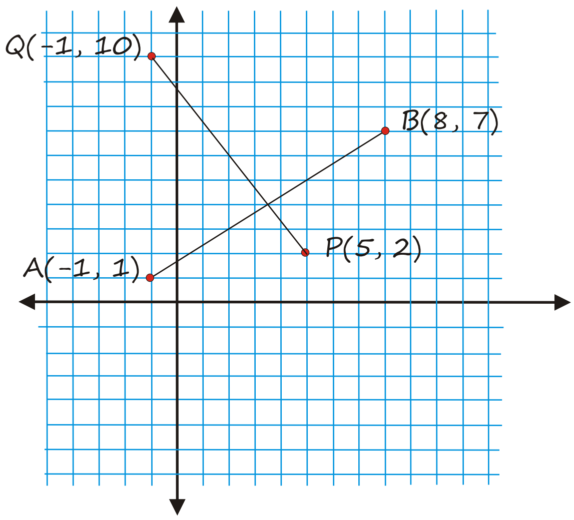
**8a.**




**8b.** Gradient AB $\dfrac{1 - 7}{-1 - 8} = \dfrac{2}{3}$

Using (−1, 1) equation $y - y_1 = m(x - x_1)$

$y - 1 = \frac{2}{3}(x + 1)$

$3y - 3 = 2(x + 1)$

$3y - 3 = 2x + 2$

$3y = 2x + 5$

**8c.** Gradient PQ $\dfrac{2 - 10}{5 + 1} = \dfrac{-4}{3}$

Using (5, 2) $y - y_1 = m(x - x_1)$

$y - 2 = \frac{-4}{3}(x - 5)$

$3y - 6 = -4(x - 5)$

$3y - 6 = -4x + 20$

$3y = -4x + 26$

**8d.** Gradients AB × PQ = −1 for perpendicular.

$\frac{2}{3} \times \frac{-4}{3} = \frac{-8}{9}$

Therefore not perpendicular

**8e.** Distance AB (−1, 1), (8, 7)

$= \sqrt{(-1 - 8)^2 + (1 - 7)^2}$

= 10.8167 (1 leg)

= 64.9 nautical miles (6 legs) 1 dp

**8f.** Distance PQ (5, 2), (−1, 10)

$= \sqrt{(5 + 1)^2 + (2 - 10)^2}$

= 10 (1 leg)

= 60 nautical miles (6 legs)

Course AB is longer by 4.9 nautical miles.

**8g.** Midpoint AB (−1, 1) (8, 7)

$\dfrac{-1 + 8}{2} \quad \dfrac{1 + 7}{2} = (3.5, 4)$

Midpoint PQ (5, 2) (−1, 10)

$\dfrac{5 - 1}{2} \quad \dfrac{2 + 10}{2} = (2, 6)$

**8h.** Equations $3y = 2x + 5$

$3y = -4x + 26$

$2x + 5 = -4x + 26$

$6x = 21$

$x = 3.5$

$y = 4$

**9a.** Total distance

$AB = 9\sqrt{5}$ $\quad \sqrt{(-12 - 6)^2 + (12 - 3)^2}$

$BC = 4\sqrt{5}$ $\quad \sqrt{(6 - 2)^2 + (3 + 5)^2}$

$CD = 4\sqrt{5}$ $\quad \sqrt{(2 + 6)^2 + (-5 + 1)^2}$

$DA = \sqrt{205}$ $\quad \sqrt{(-6 + 12)^2 + (-1 - 12)^2}$

Total Distance = 52.33

Each unit = 100m (as $100^2$ = 10 000)

Therefore 52.33 × 100 = 5233m

= 5.2 km

**9b.** Midpoint BC $\dfrac{6 + 2}{2}, \dfrac{3 - 5}{2} = (4, 1)$

Midpoint DA $\dfrac{-6 - 12}{2} \quad \dfrac{-1 + 12}{2} = (-9, 5.5)$

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**9.c.** Gradient AB (−12, 12) (6, 3)

$\frac{12 - 3}{-12 - 6} = -\frac{1}{2}$

Gradient CD (2, −5)(−6, −1)

$\frac{-5 + 1}{2 + 6} = -\frac{1}{2}$

Both gradients are the same therefore yes they are parallel.

**9d.** Gradient BC $\frac{3 + 5}{6 - 2} = 2$

Gradient AD $\frac{12 + 1}{-12 + 6} = -\frac{13}{6}$

Equation AB (6, 3), (2, −5)

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 6)$

$y = 2x - 9$

Equation BC (−12, 12), (−6, −1)

$y - y_1 = m(x - x_1)$

$y - 12 = \frac{-13}{6}(x + 12)$

$6y - 72 = -13(x + 12)$

$6y - 72 = -13x - 156$

$6y = -13x - 84$

Intersection

$y = 2x - 9$ and $6y = -13x - 84$

$6(2x - 9) = -13x - 84$

$12x - 54 = -13x - 84$

$25x = -30$

$x = -1.2$

Using $y = 2x - 9$ and $x = -1.2$

$y = -11.4$

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10.a. F(−2, 10), G(10, 2), H(−8, 6)

Using the distance formula

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

FG = 14.4222

FH = 7.2111

GH = 18.4391

All the lengths are different.

Therefore the triangle is scalene.

**10b.** Find the perpendicular bisector of all the sides.

FG $\frac{-2 + 10}{2}$ $\frac{10 + 2}{2}$ = (4, 6)

FH $\frac{-2 - 8}{2}$ $\frac{10 + 6}{2}$ = (−5, 8)

GH $\frac{10 - 8}{2}$ $\frac{2 + 6}{2}$ = (1, 4)

Find the gradients and gradients of the perpendiculars.

| | | Gradient | Perp |
|---|---|---|---|
| FG | $\frac{10 - 2}{-2 - 10}$ | $= -\frac{2}{3}$ | $\frac{3}{2}$ |
| FH | $\frac{10 - 6}{-2 + 8}$ | $= \frac{2}{3}$ | $\frac{-3}{2}$ |
| GH | $\frac{2 - 6}{10 + 8}$ | $= -\frac{2}{9}$ | $\frac{9}{2}$ |

Use the gradients of the perpendiculars to and the midpoints to find the equations.

Perpendicular Bisectors

FG
$y - y_1 = m(x - x_1)$

$y - 6 = \frac{3}{2}(x - 4)$

$2y - 12 = 3(x - 4)$

$2y - 12 = 3x - 12$

$2y - 3x = 0$

FH
$y - y_1 = m(x - x_1)$

$y - 8 = \frac{-3}{2}(x + 5)$

$2y - 16 = -3(x + 5)$

$2y - 16 = -3x - 15$

$2y + 3x = 1$

GH
$y - y_1 = m(x - x_1)$

$y - 4 = \frac{9}{2}(x - 1)$

$2y - 8 = 9(x - 1)$

$2y - 8 = 9x - 9$

$2y - 9x = -1$

Using the DS-742ET calculator simultaneous equation solver the intersection point is: (0.167, 0.25)



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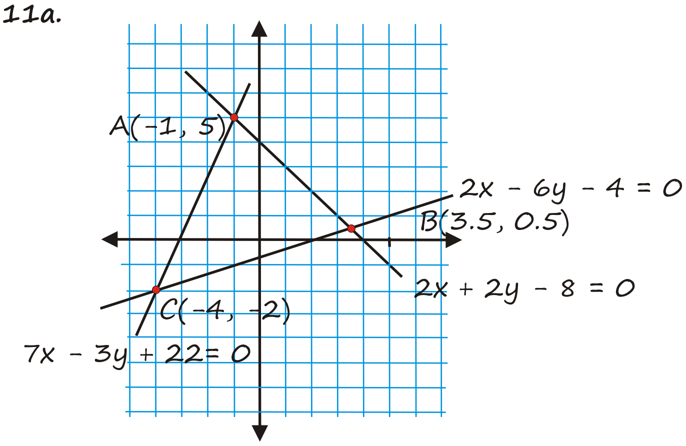
**Page 22 (cont)**

**11a.**




This question can be answered quickly by using the DS-742ET calculator and the simultaneous equation solver or you can do it the long way (below).

$2x = 6y + 4$ and $2x = -2y + 8$

$6y + 4 = -2y + 8$

$8y = 4$

$y = 0.5$

Using $2x = 6y + 4$ and $y = 0.5$

intersection point is $(3.5, 0.5)$.

$2x - 6y - 4 = 0$ or $6y = 2x - 4$

$7x - 3y + 22 = 0$

or $14x - 6y + 44 = 0$

and $6y = 14x + 44$

Therefore $2x - 4 = 14x + 44$

$-12x = 48$

$x = -4$

Using $6y = 2x - 4$ and $x = -4$

Intersection point is $(-4, -2)$

$2x + 2y - 8 = 0$ or $6x + 6y - 24 = 0$

$7x - 3y + 22 = 0$ or $14x - 6y + 44 = 0$

Therefore $6y = -6x + 24$

and $6y = 14x + 44$

$-6x + 24 = 14x + 44$

$-20x = 20$

$x = -1$

Using $2x + 2y - 8 = 0$ and $x = -1$

Intersection point is $(-1, 5)$

**11b.** The orthocentre of a triangle is found when the 3 altitudes all meet at one point. Therefore the first task is to find the gradients of each line.

$2x - 6y - 4 = 0$, gradient $= \frac{1}{3}$

$2x + 2y - 8 = 0$, gradient $= -1$

$7x - 3y + 22 = 0$, gradient $= \frac{7}{3}$

The gradients of the perpendiculars

$m1m2 = -1$

Therefore the perpendicular gradients and the triangle vertex point opposite are:

| Triangle Vertex | Gradient |
|---|---|
| A (−1, 5) | −3 |
| C (−4, −2) | 1 |
| B (3.5, 0.5) | $\frac{-3}{7}$ |

Equations of perpendiculars

using $y - y_1 = m(x - x_1)$

Point A: $y - 5 = -3(x + 1)$

$y - 5 = -3x - 3$

$y = -3x + 2$

Point C: $y + 2 = 1(x + 4)$

$y = x + 2$

Point B: $y - 0.5 = \frac{-3}{7}(x - 3.5)$

$y - 0.5 = \frac{-3}{7}x + \frac{3}{2}$

$y = \frac{-3}{7}x + 2$

$7y = -3x + 14$

Intersection points:

$y = -3x + 2$ and $y = x + 2$

$-3x + 2 = x + 2$

$-4x = 0$

$x = 0$ and $y = 2$

$y = x + 2$ and $7y = -3x + 14$

$7(x + 2) = -3x + 14$

$7x + 14 = -3x + 14$

$10x = 0$

$x = 0$ and $y = 2$

$y = -3x + 2$ and $7y = -3x + 14$

$7(-3x + 2) = -3x + 14$

$-21x + 14 = -3x + 14$

$-18x = 0$

$x = 0, y = 2$

Therefore the orthocentre is $(0, 2)$

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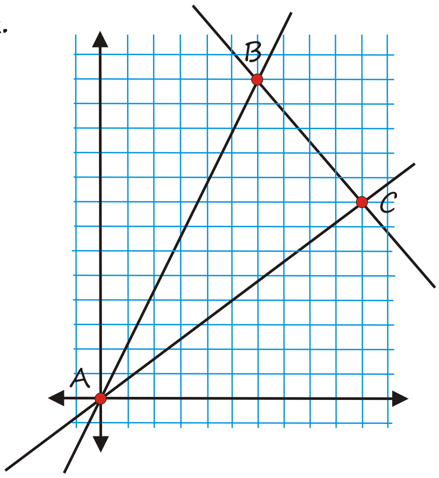
**Page 22 (cont)**

**12a.**




Gradient AB (0, 0), (6, 13)

$\frac{0 - 13}{0 - 6} = \frac{13}{6}$

Equation $y - y_1 = m(x - x_1)$ using (0, 0)

$y = \frac{13}{6}x$ or $6y = 13x$

Gradient AC (0, 0), (10, 8)

$\frac{0 - 8}{0 - 10} = \frac{4}{5}$

Equation $y - y_1 = m(x - x_1)$ using (0, 0)

$y = \frac{4}{5}x$ or $5y = 4x$

Gradient BC (6, 13), (10, 8)

$\frac{8 - 13}{10 - 6} = \frac{-5}{4}$

Equation $y - y_1 = m(x - x_1)$ using (10, 8)

$y - 8 = \frac{-5}{4}(x - 10)$

$4y - 32 = -5x + 50$

$4y = -5x + 82$

**b.** The midpoints should be as follows.

Midpoint AB x(3, 6.5)

Midpoint AC y(5, 4)

Midpoint BC z(8, 10.5)

**c.** Notice how the gradients of AC and BC are perpendicular.

Therefore the area of triangle = ½AC × BC

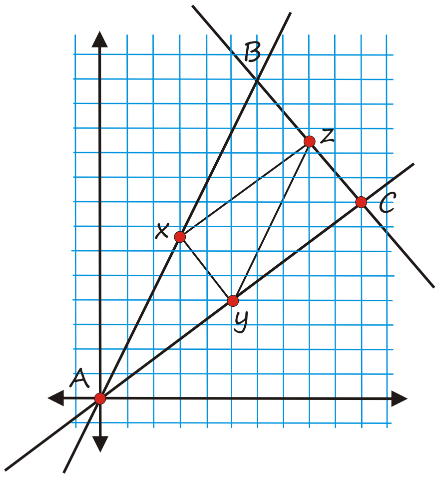
Lengths of AC and BC using

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$AC = 2\sqrt{41}$, $BC = \sqrt{41}$

Therefore Area = 41 (units²)

**12c. (continued)**




x(3, 6.5), y(5, 4), (8, 10.5)

xy is perpendicular to xz

This is because the gradient of xy $= \frac{-5}{4}$

and the gradient of xz $= \frac{4}{5}$

Therefore the area of triangle = ½xy × xz

Using $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

length $xy = \frac{\sqrt{41}}{2}$ $xz = \sqrt{41}$

Area $xyz = \frac{\sqrt{41}}{4}$

Therefore using this example the calculation seems to indicate that the theory is correct.

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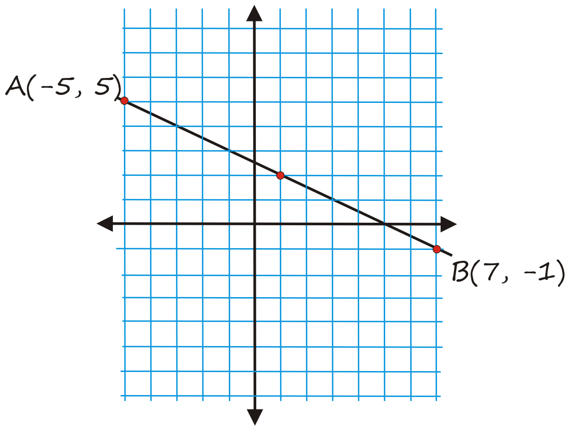
**Page 22 (cont)**

**13.**




When you look at the graph of AB above you will note that an isosceles triangle could be formed at the top of the line or at the bottom of the line. Therefore there will be two possible answers.

Length of AB = $6\sqrt{5}$

Gradient of AB

$$\frac{-1-5}{7+5} = \frac{-1}{2}$$

Midpoint of AB $\left(\frac{-5+7}{2}, \frac{5+-1}{2}\right) = (1, 2)$

Gradient of perpendicular to AB = 2

Therefore equation of the line perpendicular to AB and through the point (1, 2) is:

$y - 2 = 2(x - 1)$

$y = 2x$

Call the point C(x, y)

Therefore Area = ½AB × height

$30 = \frac{1}{2} \times 6\sqrt{5} \times \text{height}$

$\text{height} = 2\sqrt{5}$

This means the distance between the midpoint (1, 2) and (x, y) must be $2\sqrt{5}$

Using $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$2\sqrt{5} = \sqrt{(1 - x)^2 + (2 - y)^2}$

$20 = (1 - x)^2 + (2 - y)^2$

Using $y = 2x$

$20 = (1 - x)^2 + (2 - 2x)^2$

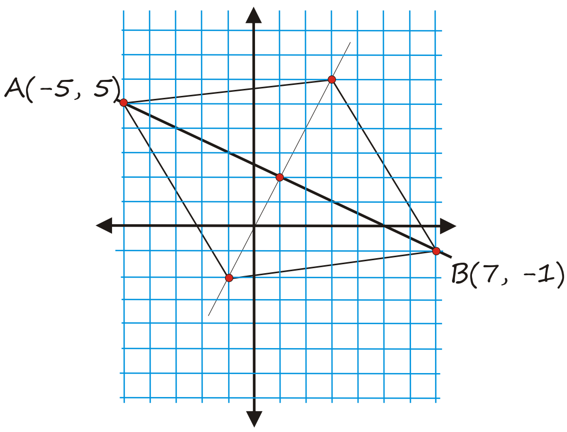
$20 = 1 - 2x + x^2 + 4 - 8x + 4x^2$

**13 (cont)**

$5x^2 - 10x - 15 = 0$

This means $x = 3$ or $x = -1$

Using $y = 2x$ the possible points are (3, 6) or (–1, –2)



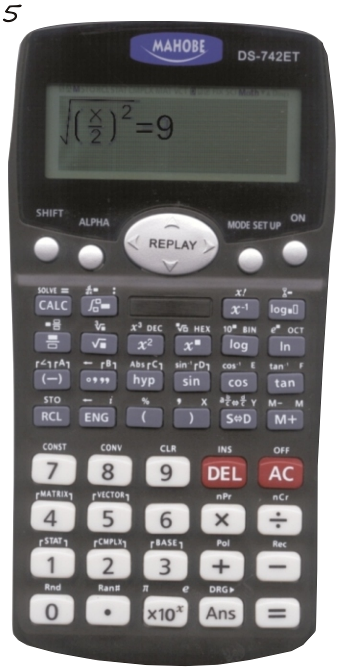

You could double check your answer by finding the length between the mid point (1, 2) and (3, 6). Using the distance formula the distance is $2\sqrt{5}$

Using the area of a triangle

½ × base × height

$= \frac{1}{2} \times 6\sqrt{5} \times 2\sqrt{5}$

= 30 units²




The New Zealand Centre of Mathematics recommends the DS-742ET calculator. It shows equations the same way they are written in textbooks and it has a built in equation solver. Calculating coordinates becomes more fun when you have a good calculator.

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