SIMULATIONS CHANCE AND DATA

Kim Freeman

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About This Book

In a 2011 study at the 'Institute for the Future' at the University of Phoenix, researchers identified 10 skills for the future workforce leading to the year 2020. Skill number 5 was "computational thinking". Our exposure to data is increasing at an exponential rate and the institute noted that the use of simulations will become a core expertise and skill in helping to make decisions. Being able to analyse data, choose the best model for approximations and being aware of limitations will become increasingly more important skills.

The activities and investigations in this book help to develop and reinforce many of the basic concepts of probability. Simulations are often used when it is difficult to calculate the theoretical answer but most of these activities do come with the theory explained. At school, the use of calculators and computer simulation programs are recommended rather than flipping coins or rolling dice many times. However sometimes access to computers can be difficult. Some students will prefer to download coin and dice apps for their phones. These can be slow when doing large numbers of trials however it is up to each individual student to decide whether or not they are appropriate to use. Spreadsheet and computer programs can quickly demonstrate to students central tendency and they should come to realise that using means from several smaller sets will vary more than using the mean of a large data set. Therefore both junior and senior students should realise that increasing the sample size will increase the reliability of the results. Let them come up with the notion of combining trial results to help increase the reliability of the simulation data. In many class situations computers are not always accessible, therefore having each student perform 20 or 30 trials and then combining the results with others is still acceptable and fun.

After each activity of simulations, students are guided toward the theoretical answer. While most activities get them to work independently they are encouraged to compare their answers with others. Teachers should be able to help with any analysis and with most of the activities teacher notes are included.

At Year 10 students need to be able to describe the distribution of the trials and write what they think would be reasonable ranges from the trial data. The theoretical probabilities are quite often solved by using probability trees. To do this students will need to be able to use fractions and ratios. At year 11 students need to be able to design their own simulations and they are assessed on their ability to describe and simply analyse what has happened. At year 12 students need to carry out a 1 or 2 step simulation. They are assessed on their ability to describe and carry out the simulation and link their results to the theoretical probability.

The use of these simulations leads naturally to permutations, combinations, conditional probability and expected values. For senior classes there is also a natural progression to normal distribution and confidence interval work.

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RND#

The Random Number (RND#) function is found on both the Mahobe SPYDER and the Mahobe DS-742ET scientific calculators. A three digit decimal number can be generated by first pressing the **SHIFT** key then **RND#** and **=**.

The result is a random 3 digit decimal number between 0 and 1.

To emulate a dice throw (1, 2, 3, 4, 5, or 6) then you would key in 1 + 6 **RND#**

In theory 6 **RND#** (6 times the random number) widens the possible output to between 0 and 6. However in reality the smallest number **RND#** will produce is 0.001 and the largest number is 0.999. This means multiplying each of these by 6 produces a result between 0.006 and 5.994. This is why you add 1. It means the possible output will be now be inclusively between 1.006 and 6.994. When noting the results for a simulation you only need to note the integer part of the random number. Ignore the decimal places (i.e. truncate the result).

Here are some typical scenarios for using the **RND#** function:
In each scenario remember that you have to truncate the result.

1. To generate random numbers 1 or 2 key 1 + 2 **RND#**

2. To generate random numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10: key 1 + 10 **RND#**

3. To simulate the New Zealand Lotto game: key 1 + 40 **RND#**

4. To simulate a 20% chance of success in a situation key 100 **RND#**
Truncate the result and use 00 to 19 for success and 20 to 99 for failure.
For such an example you could also just use the 3 digit number produced.

Other useful calculations that teachers should remember:

Long-run relative frequency = number of successful trials ÷ number of trials

Mean = sum of the trial results ÷ number of trials

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Using Excel

Excel has great potential for students to build simple or complex simulations with only a few formula commands. Three of the main ones are explained below.

=RANDBETWEEN(bottom,top**)**
This command returns an integer between the two numbers that you specify.

e.g. =RANDBETWEEN(1,6) will give possible results 1, 2, 3, 4, 5, 6.

=IF(logical_test, [value if true], [value if false]**)**
This command checks to see if a condition is met and returns one value if true and another value if false.

e.g. = IF(A1 < 10, “result”, “no result”)
If the number in cell A1 is less than 10 then the word “result” is printed in the cell. If the number is 10 or greater then the words “no result” are printed. When you want words or letters to be displayed then speech marks have to be around the word. They are not needed for numbers to be displayed.

e.g. If you wanted to count the number of 6s when throwing a die you could use the command =RANDBETWEEN(1, 6) in the first column of cells. You could then use the command =IF(A1 = 6, 1, 0). This would print a 1 if the number was a 6 and a 0 is the number was a 1, 2, 3, 4 or 5. To then count the number of 6s in the simulation you would just need to sum the column of 0 and 1s.

You can “nest” IF statements. For example if you wanted to simulate success being odd numbers thrown with a die.

e.g. =IF(A1 = 1, 1, IF(A1 = 3, 1, IF(A1 = 5, 1, 0)))
In this statement each of the false values test another condition. If A1 contains a 2, 4 or 6 then the result will be a 0. If A1 contains a 1, 3 or 5 then the result will be a 1 printed in the cell.

=COUNTIF(range, criteria**)**
This can be used as an alternative to the IF statement. If counts the number of cells within a range that fits a certain criterial.

e.g. =COUNTIF(A1..A100, 5)
This statement scans all the cells between A1 and A100 and counts all that have a 5 in them.

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TEACHER NOTES

HOW MANY TIMES AND WINNING STREAKS

In this set of simulations students are asked to investigate how many throws of a coin they need to perform before you have the same number of heads as tails. The second part of the exercise then looks at winning streaks. If two teams are seemingly evenly matched then why do some win more often than others. This could be applied to such things as rugby games. If a team has won the last few encounters does that mean that they will forever win? How can we tell statistically if one team is better than others?

Students will be surprised at how easy it is to have winning streaks for 3, 4 or more games. There are a number of good questions for them to investigate such as: does a better player have more or longer winning streaks? Question 10 covers this with some extra investigations. Do more winning streaks or longer winning streaks suggest a player or team is better? When investigating this, students should find that as the probability of winning goes above 0.5 the number of winning streaks goes down but their average length goes up. Similarly, as the number of winning streaks goes below 0.5, the number of winning streaks goes down and their average length goes down. Therefore number of streaks and their average length depends on the chance or probability of winning.

The Advanced Theory
Tossing coins and predicting their result is a binomial distribution as each game has two possible outcomes. However here we are more concerned with the expected number of winning streaks i.e. the number of wins after a loss. If two players are equally likely to win then the probability that the game is the first of a winning streak is calculated as below. Note - a winning streak starts after a loss.

Loss × Win = $0.5 \times 0.5 = (0.5)^2 = 0.25$

If the streak is only 1 game long then the sequence is Loss × Win × Loss
$0.5 \times 0.5 \times 0.5 = 0.125$

If the streak is 2 games long then the sequence is Loss × Win × Win × Loss
$0.5 \times 0.5 \times 0.5 \times 0.5 = 0.0625$

Therefore in 100 games you can expect 25 winning streaks that are made up of the following lengths:

Streak Length	1	2	3	4	5
Frequency	12 or 13	6 or 7	3	1 or 2	1

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If the probability of winning is increased then the probabilities change as below. If the probability of winning is now 0.75 then the chance of winning is

$0.25 \times 0.75 = 0.1875$

If the streak is only 1 game long then the probability is:
$0.25 \times 0.75 \times 0.25 = 0.0469$

If the streak is 2 games long then the probability is:
$0.25 \times 0.75 \times 0.75 \times 0.25 = 0.0352$

Therefore in 100 games you can expect 18 or 19 winning streaks that are made up of the following lengths:

Streak Length	1	2	3	4	5
Frequency	4 or 5	3 or 4	2 or 3	1 or 2	1

The probability of a winning streak can be shown with the formula:

Pr(winning streak of n times) = $p^n \times (1 - p)^2$ where p = winning probability.

The formula for the average length of winning streaks is: $\frac{1}{1 - p}$.

The total streaks in n games can be calculated by: $np(1 - p)$.

Using these formulas and 100 game simulation the expected values are shown.

	Probability of winning								
streak length	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	8.1	12.8	14.4	14.7	12.5	9.6	6.3	3.2	0.9
2	0.8	2.6	4.4	5.8	6.3	5.8	4.4	2.6	0.8
3	0.1	0.5	1.3	2.3	3.1	3.5	3.1	2.1	0.7
4	0.0	0.1	0.4	0.9	1.6	2.1	2.2	1.6	0.7
5	0.0	0.0	0.1	0.4	0.8	1.2	1.5	1.3	0.6
Average length	1.1	1.3	1.4	1.7	2	2.5	3.3	5.0	10.0
Streaks in 100 games	9	16	24	21	25	24	21	16	9

The table is included for better or more advanced students who work on question 10. The theory that a team continues to have the same probability of winning through 100 games may not be realistic. The use of this table can lead to a discussion on how to predict sports game results however the probability of any team winning depends on the relative strengths of each team in each game.

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HOW MANY TIMES and WINNING STREAKS

When you toss a coin it can land as a "Head" or a "Tail". Mathematically this means that you have 1 chance out of 2 (written as ½) or a 50% chance of getting either a head or a tail.

1. When it comes to chance and one-off-events, theoretical probability and what actually happens can be quite different. Estimate how many throws of a coin you think you would need to perform before you have the same number of heads as tails?

Estimate . . . .

2. Test to see if your estimate is correct. Using 10 trials, throw a coin and record the results. Stop each trial after you have the same number of heads as tails. After 10 trials write the mean number of throws needed.

	results	Total throws
Example Trial	H H T H T T	6
Trial 1		
Trial 2		
Trial 3		
Trial 4		
Trial 5		
Trial 6		
Trial 7		
Trial 8		
Trial 9		
Trial 10		
		Mean

3. What was the longest winning streak of heads and tails? . . . .

4. Using the results from at least 6 others students in your class find the least, the most and the mean number of throws needed before 50% heads and 50% tails was achieved?

Number of Throws Least . . . . . . Mean . . . . . . Most . . . . . .

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You play Dorothy each week at tennis. You both play in the same grade for your club and consider yourselves of equal ability. In your games together you both seem to win an equal amount of the time. However in one particular period Dorothy wins five games in a row before you can beat her again. Can you or Dorothy claim to be the better player?

5. In the simulation below you will find how common different winning streaks are if there are two teams or two people of equal ability. Either use a coin or a die (using heads / tails, odds / evens or 1, 2 or 3 for a win and 4, 5 or 6 for a loss). Play 100 games and record the results in the table below as a W (win) or L (loss).

A winning streak is defined as the number of wins in a row before a loss.
A winning streak can be 1, 2, 3, 4 or more.

6. Analysing the outcomes.

The total number of wins was . . . .

The total number of losses was . . . .

The total number of winning streaks was . . . .

The average winning streak length was . . . .

Record your results as well as 4 others in the table below.

	Number of Wins	**Number of Streaks**	**Length of Wining Streak**						
			1	**2**	**3**	**4**	**5**	**6**	**7**
i.									
ii.									
iii.									
iv									
v.									
Averages									

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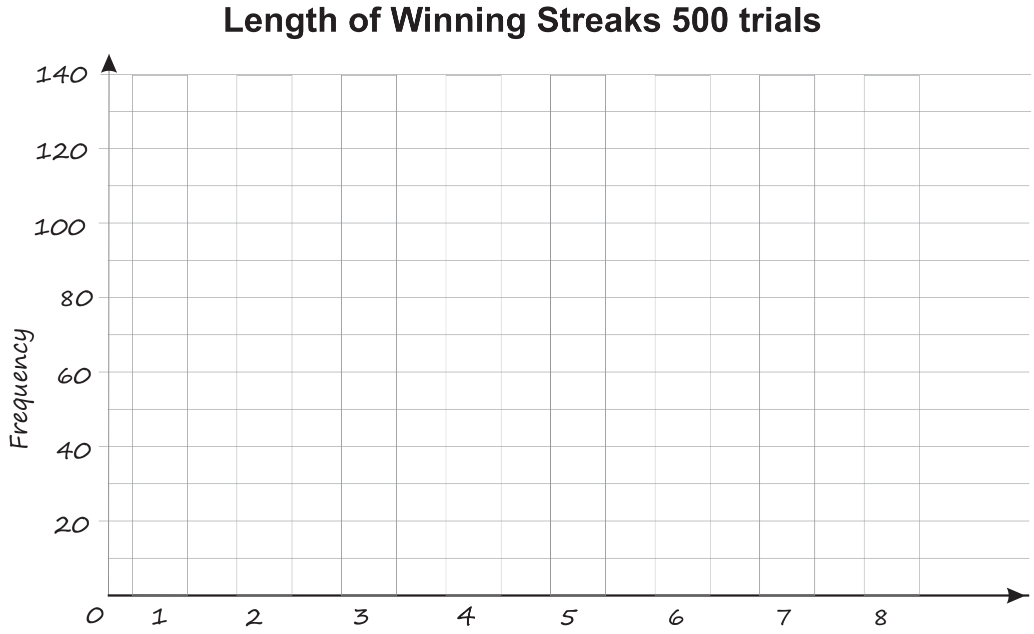
7. Draw a bar graph that shows the number of each winning streak size.



8. Even if players or teams are of seemingly equal ability, it is still possible for one to have a 6 or 7 game winning streak. How then can you tell if one player is better than another? Is it because they have longer or more winning streaks or is it because of some other factor?

. . . .

9. Suppose a player has a probability of winning other than 50% (e.g 10%, 20%, 30%, 40%, 60%, 70%, 80% or 90%). Divide your class into groups and design a 100 game simulation with each group choosing a different probability. Make a summary of the results on your own paper.

10. Using the results from question 9 answer the following questions.

i. As the probability goes above 0.5 the number of winning streaks goes

. . . . and their average length goes . . . .

ii. As the probability goes below 0.5 the number of winning streaks goes

. . . . and the average length goes . . . .

iii. From this experiment how can you tell if one player or team is better than another?

. . . .

. . . .

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RND#
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SUMS AND DIFFERENCES

When you toss a die the chance of throwing a 1, 2, 3, 4, 5 or 6 is $\frac{1}{6}$.

However how many times do you think you would need to throw a die before each number has appeared one sixth of the time?

Estimate: . . . .

1. Each member of the class should throw a die and record the results. Stop when you have an equal amount of each number. Record your results in the table below. Then compare your results with others in your class.

1	2	3	4	5	6

2. Using the results from your whole class give the following statistics:

Minimum number of throws . . . . . .

Maximum number of throws: . . . . .

Median number of throws: . . . . . .

Mean number of throws . . . . . .

3. In this game you can win if you throw a 1, 2, 3, 4, and lose if you throw a 5 or 6. This means you have 4 out of 6 chances (or 2/3) of winning. How many times do you need to throw the die until you have won 2/3 of the time?

Win

Lose

4. Compare your results with others in the class. If you were going to conduct an experiment where you didn't know the probabilities of each value occurring, how many trials would you use to be sure that you had the best result?

. . . .

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When you throw two dice the difference between the two is 0, 1, 2, 3, 4 or 5.

5. Choose a partner.

Player 1 wins if the difference between the two numbers is 0, 1 or 2.

Player 2 wins if the difference between the two numbers is 3, 4 or 5.

Who do you think will win? Give a reason for your answer.

. . . .

. . . .

Toss the two dice 60 times and record the results of who wins.

6.

Results	Number of Wins	Total /60
Player 1: 0, 1, 2		
Player 2: 3, 4, 5		

7. Now record your results along with 9 others in your class.

											Total
Player 1:											
Player 2:											

8. Below is a table of all the theoretical results. Complete the table and highlight all the possible differences for Player 1 to win.

die 1

die 2	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1					
3	2					
4	3					
5	4					
6	5					

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9. Was the game fair?

. . . .

10. What is the theoretical probability that each player wins? How close were each of the results to the actual theoretical probability?

. . . .

11. How can we modify this game to make it fair?

. . . .

12. In this next game simulation, Player 1 wins if the sum of the two dice is 5, 6, 7 or 8 and Player 2 wins if the sum is 2, 3, 4, 9, 10, 11 or 12.

Who do you think will win the most? . . . .
Toss the dice 60 times and record who wins.

Results	Number of Wins	Total /60
Player 1: 5, 6, 7, 8		
Player 2: 2, 3, 4, 9, 10, 11		

13. Now record your results along with 9 others in your class.

											Total
Player 1:											
Player 2:											

14. Was this game fair? Write down the theoretical probability of winning for each player and compare it to the results from your class. Finally, choose sums that will give each player an equal and fair chance of winning.

. . . .

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TEACHER NOTES

FAMILY MATTERS

In this set of simulations students are asked to investigate the compositions of different sized families. They could be used at Year 10, 11 or 12. In each of these simulations students are guided through the steps and make some interpretations of the results. The simulations are designed so that students can identify how each particular scenario can have a strategy designed to explore each of the possible probabilities.

In most cases a simulation of 50 trials is suggested. If they want to extend the trials then each student could combine their results with another as 100 trials makes the probability calculations a little easier. The second simulation exercise also has students thinking about the ideal number of trials.

More trials lead to a demonstration of central tendency and students should come to realise that using several smaller sets will vary more than using the means of large data sets. Both junior and senior students should realise that increasing the sample size increases the reliability of the results.

In most cases a coin, dice or scientific calculator can be used. However iPhones and Android phones both have coin and dice apps that can be downloaded and used.

In Year 11 students are expected to design and describe the simulation as well as produce a reasonable estimate for the probability. In more advanced simulations at Year 12, students are expected to link their results to the actual theory or compare their results to another simulation.

Scenarios in this series of activities include:

1 son families

2 child families

3 child families

4 child families

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FAMILY MATTERS - 1 son

In some countries parents look to their son to take care of them when they are old. Therefore they quite often keep having children until a son is born. Some governments discourage this as it can result in over population.

Carry out a 50 trial simulation to estimate the average family size if parents are allowed to continue having children until a male is born. To do this you could use the RAN# (Random Number) function on your calculator, flip a coin or use dice.

1. Write your method here: . . . .

2. Carry out 50 trials and record your results in the table below.

Trial	Outcome		Trial	Outcome		Trial	Outcome	
e.g.	M	1	e.g	F F M	3	e.g	F M	2
1.			18.			35.		
2.			19.			36.		
3.			20.			37.		
4.			21.			38.		
5.			22.			39.		
6.			23.			40.		
7.			24.			41.		
8.			25.			42.		
9.			26.			43.		
10.			27.			44.		
11.			28.			45.		
12.			29.			46.		
13.			30.			47.		
14.			31.			48.		
15.			32.			49.		
16.			33.			50.		
17.			34.					

$$\text{Average family size} = \frac{\text{sum of each family size}}{\text{50 (number of trials)}}$$

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FAMILY MATTERS - 2 child families

In this simulation you are asked to answer one of the following questions. Put a tick beside the question that you are going to try and answer.

a. What the chances are of having 1 boy and 1 girl in a family of 2 children?

b. Are there equal chances of having 2 boys as there are of having 2 girls or 1 boy and 1 girl in a family of 2 children?

You are also going to investigate whether more trials in a simulation gives a more accurate result. To answer your question you are going to carry out a simulation of 50 trials. Because there are only 2 possible choices you can flip a coin to simulate the sex of each of the 2 children, you could use the RND# (Random Number) function on your calculator or a die with 1,2,3, or odd/even numbers representing either sex.

1. Write your method below.

2. Carry out 50 trials and record your results in the table.

e.g. M M	M F	F M
1.	18.	35.
2.	19.	36.
3.	20.	37.
4.	21.	38.
5.	22.	39.
6.	23.	40.
7.	24.	41.
8.	25.	42.
9.	26.	43.
10.	27.	44.
11.	28.	45.
12.	29.	46.
13.	30.	47.
14.	31.	48.
15.	32.	49.
16.	33.	50.
17.	34.	

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The probability that there are 2 males, 2 females or 1 male and 1 female in a family of 2 can be calculated by using the results of the simulation and the following formula:

$$\frac{\text{number of successful results}}{\text{number of trials}}$$

3. Use this formula and your results to calculate the following probabilities:

Probability of 2 boys . . . .

Probability of 1 girl and 1 boy . . . .

Probability of 2 girls . . . .

In this next section you are going to observe whether having more trials in your simulation is better for getting a more accurate result.

4. Combine your results with one other person (100 trials), 3 other people (200 trials) and then 9 other people (500 trials). Record all the results then calculate each of the probabilities.

Class simulation results:

	BB	BG+GB	GG
1.	. . .	. . .	. . .
2.	. . .	. . .	. . .
3.	. . .	. . .	. . .
4.	. . .	. . .	. . .
5.	. . .	. . .	. . .
6.	. . .	. . .	. . .
7.	. . .	. . .	. . .
8.	. . .	. . .	. . .
9.	. . .	. . .	. . .
10.	. . .	. . .	. . .

Probabilities

$$\frac{\text{number of successful results}}{\text{number of trials}}$$

Trials	BB	BG+GB	GG
100.	. . .	. . .	. . .
200.	. . .	. . .	. . .
500.	. . .	. . .	. . .

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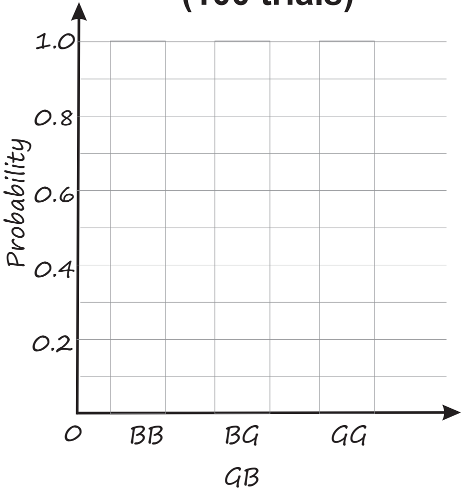
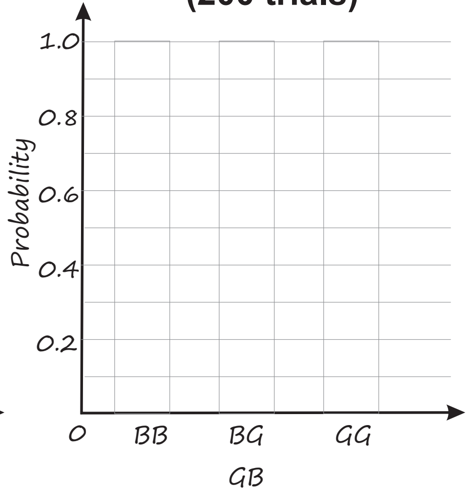
5. Using your probabilities complete the graphs below.

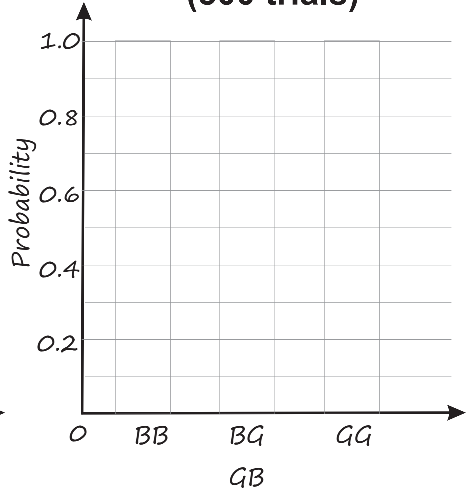
Composition of a 2 Child Family (100 trials)



Composition of a 2 Child Family (200 trials)



Composition of a 2 Child Family (500 trials)



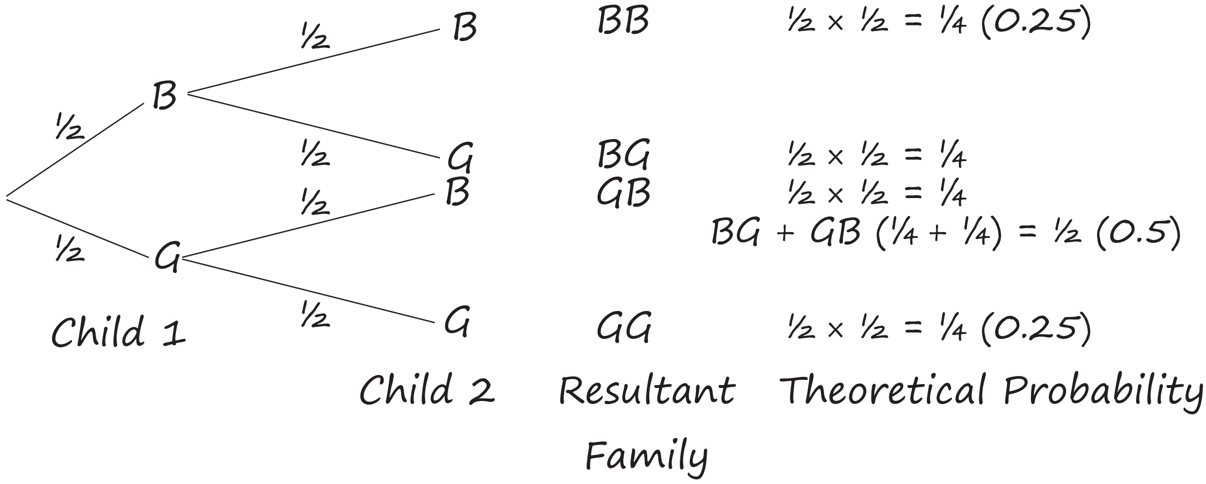
6. As the number of trials increased what happened to each of the probabilities. Is there an ideal number of simulation trials?

Explain your answer.

. . . .

. . . .

The tree diagram below gives shows the theoretical probabilities for each combination of children in a 2 child family.



7. Write a conclusion that answers the original question you chose.

. . . .

. . . .

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MAHOBE
Scientific Calculator
RND#
0.791

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FAMILY MATTERS - 3 child families

Justine comes from a family with 3 children. She has 2 sisters meaning that all the children in her family are girls. However her best friend, Paris, has 2 brothers and no sisters. In this task you are to carry out a simulation to find the chance that a family of 3 children will be made up of: 2 boys and a girl, 2 girls and a boy, 3 girls or 3 boys. To do this you could use the RAN# (Random Number) function on your calculator, flip a coin or use dice.

1. Write your method below.

2. Carry out 50 trials and record your results in the table below.

Trial	Outcome	Trial	Outcome	Trial	Outcome
e.g.	M F M	e.g	M F F	e.g	F M F
1.		18.		35.	
2.		19.		36.	
3.		20.		37.	
4.		21.		38.	
5.		22.		39.	
6.		23.		40.	
7.		24.		41.	
8.		25.		42.	
9.		26.		43.	
10.		27.		44.	
11.		28.		45.	
12.		29.		46.	
13.		30.		47.	
14.		31.		48.	
15.		32.		49.	
16.		33.		50.	
17.		34.			

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RESULTS Fraction

Probability of:

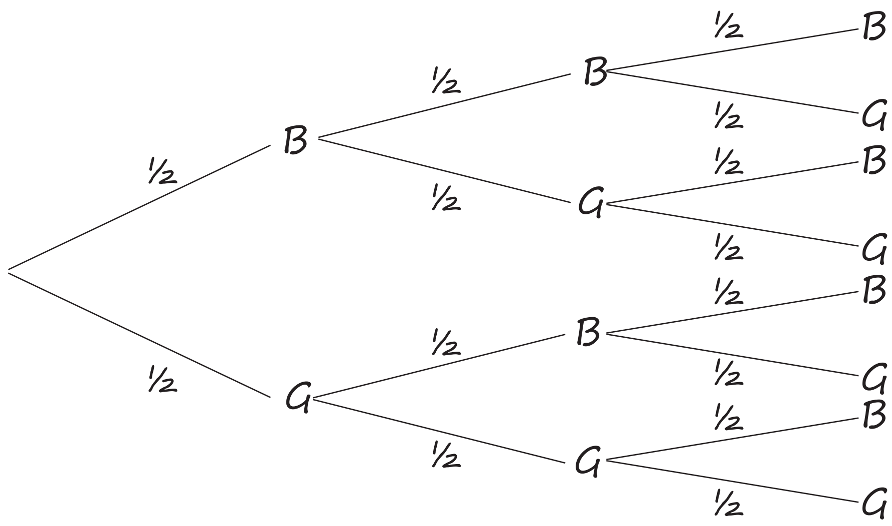
3 boys (BBB) = $\frac{\text{number of successful results}}{\text{number of trials}}$ = ___

2 boys and 1 girl (BBG, BGB, GBB) = $\frac{\text{number of successful results}}{\text{number of trials}}$ = ___

2 girls and 1 boy (GGB, GBG, BGG) = $\frac{\text{number of successful results}}{\text{number of trials}}$ = ___

3 girls (GGG) = $\frac{\text{number of successful results}}{\text{number of trials}}$ = ___

THE THEORY The probability tree shows all the possible outcomes for 3 children along with the theoretical probability that they will happen.



Outcome	Probability
BBB	½ × ½ × ½ = ⅛
BBG	½ × ½ × ½ = ⅛
BGB	½ × ½ × ½ = ⅛
BGG	½ × ½ × ½ = ⅛
GBB	½ × ½ × ½ = ⅛
GBG	½ × ½ × ½ = ⅛
GGB	½ × ½ × ½ = ⅛
GGG	½ × ½ × ½ = ⅛

3. Compare your results with the theoretical results.

Outcome	Theoretical Probability	My Results
BBB		
BBG, BGB, GBB		
GGB, BGB, BGG		
GGG		

4. How could you improve your results so that they are closer to the theoretical results and what are the advantages / disadvantages of this simulation?

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FAMILY MATTERS - 4 child families

Katherine comes from a family with 4 children. She has 1 brother and 2 sisters. Her theory is that half of families with 4 children will have exactly 2 boys and 2 girls. In this task you will test to see if her theory is correct.

1. Carry out a simulation to find the most likely composition of a 4 child family. To do this you could use the RAN# (Random Number) function on your calculator, flip a coin or use another method. Write your method below.

2. Carry out 50 trials and record your results in the table below.

Trial Outcome	Trial Outcome	Trial Outcome
e.g. M F M F	e.g M F F F	e.g F M F M
1.	18.	35.
2.	19.	36.
3.	20.	37.
4.	21.	38.
5.	22.	39.
6.	23.	40.
7.	24.	41.
8.	25.	42.
9.	26.	43.
10.	27.	44.
11.	28.	45.
12.	29.	46.
13.	30.	47.
14.	31.	48.
15.	32.	49.
16.	33.	50.
17.	34.	

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RESULTS Fraction

Probability of:

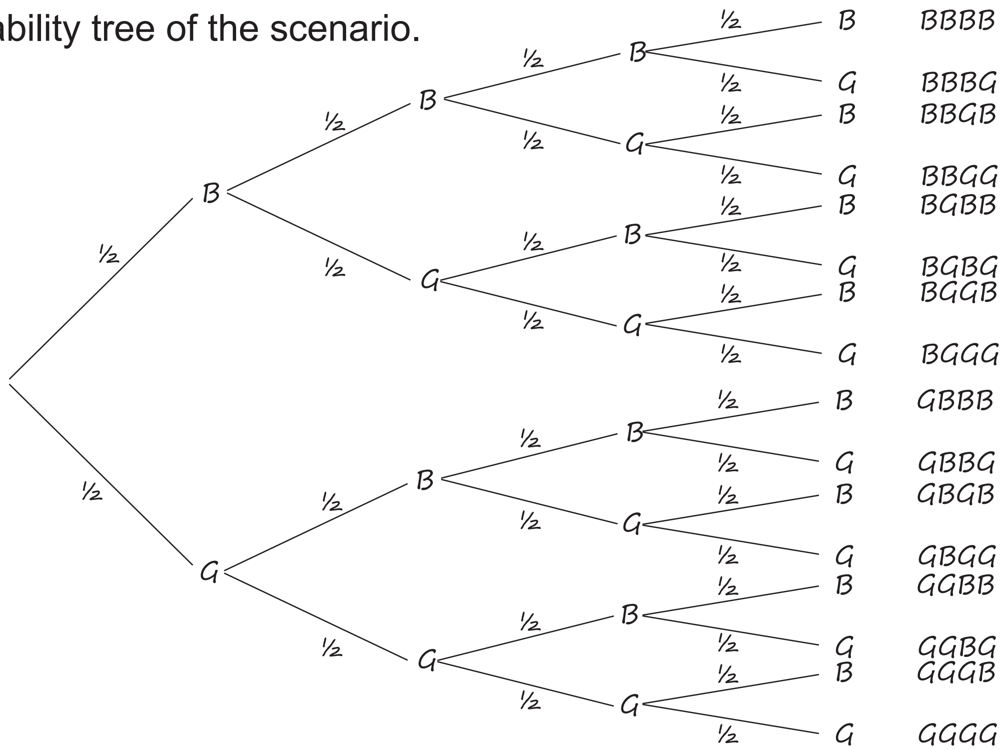
2 boys and 2 girls = $\frac{\text{number of successful results}}{\text{number of trials}}$ = ——

3. Was Katherine correct with her theory that half of 4 children families will have exactly 2 boys and 2 girls? Explain what you think the most likely probability will be.

. . . .

. . . .

Here is a probability tree of the scenario.



3. Compare your results with the theoretical results.

Outcome	Theoretical Probability	My Results
BBBB	½ × ½ × ½ × ½ = 1/16	
BBBG, BBGB, BGBB, GBBB	4 × 1/16 = ¼	
BBGG, BGBG, BGGB, GBBG, GBGB GGBB	6 × 1/16 = 3/8	
BGGG, GBGG, GGBG, GGGB	4 × 1/16 = ¼	
GGGG	½ × ½ × ½ × ½ = 1/16	

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TEACHER NOTES

MAHOBE TEA

When you use a device to produce random outcomes (such as dice, coins, spreadsheet or other computer simulation) then it is known as a “Monte Carlo Simulation”.

The next simulation can be done using dice, the RAN# function on a calculator or a spreadsheet. If there are 6 different jigsaw pieces in packets of tea how many packets would you need to purchase in order to have all 6 jigsaw pieces? In a computer simulation of 15000 trials of this experiment it was found that on average 14.68 packets of tea would have to be purchased. However overall results ranged from 8 to 49 packets.

In order to calculate the theoretical probability you need to understand the relationship between the probability of getting a certain jigsaw piece prize and the expected number of purchases to do so. If you think of the dice and the chance of getting an odd number being one half (1/2) on each roll, then you would expect to have to roll the dice an average of two times before seeing an odd number. In general, the expected value is the reciprocal of the probability.

On the first trip to purchase tea you would be happy with any of the six puzzle pieces. Since the probability of getting any piece is 6/6 then the expected value is also 6/6, or 1.

On the second day, you need 1 of 5 pieces. The probability of getting a desired jigsaw piece is 5/6 making the expected value 6/5.

Therefore the total expected purchases before getting all the jigsaw pieces is the sum: 6/6 + 6/5 + 6/4 + 6/3 + 6/2 + 6/1.

This calculates to be about 14.7, (15) packets of tea.

This is a popular simulation as quite often companies (especially breakfast cereal companies) run such promotions. Quite often the promotion only lasts for 2 or 3 months. This can lead to questions such as “will you be able to drink enough tea to actually purchase all the packets needed to get all 6 jigsaw puzzle pieces?” Is this a marketing ploy for consumers to purchase more products?

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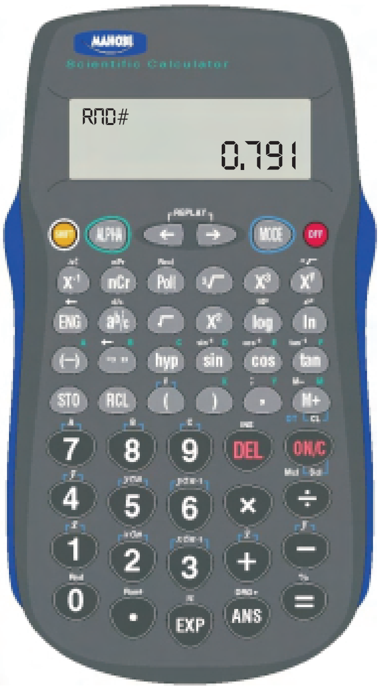

SPYDER

If buying class sets of calculators then this is the logical choice. They are easily recognisable but more importantly they are tough, reliable and contain all the functions that students need. Only the Mahobe SPYDER calculator is recommended by The New Zealand Centre of Mathematics. Sales of this calculator help bring you outstanding classroom resources. Purchase it direct from the Mahobe website:

www.mahobe.co.nz.

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MAHOBE TEA

Mahobe makes a brand of tea. Inside each box of tea bags they put the piece of a jigsaw puzzle. Once a customer has all 6 pieces of the jigsaw then they can send away for a new Mahobe teapot.

The jigsaw pieces are equally and randomly distributed with the tea boxes. The owners of the company want to know how many boxes of tea, on average, a customer needs to purchase in order to get the free teapot. The activity below is a probability experiment (simulation) to mimic this situation and therefore find the expected number of boxes sold.

Work in pairs or small groups to complete these steps.

1. Use the six numbers on a die or use your calculator's RND# function to generate numbers between 1 and 6.

2. Record which jigsaw piece is shown in the chart below.

3. Continue to roll the die or generate random numbers until you have a complete set of pieces. Stop as soon as you have a complete set as this is the end of 1 trial. Record the number of boxes required for this trial.

4. Repeat steps 1,2 and 3 until your group has carried out 25 trials**.**

Trial		Trial	
1.		14.	
2.		15.	
3.		16.	
4.		17.	
5.		18.	
6.		19.	
7.		20.	
8.		21.	
9.		22.	
10.		23.	
11.		24.	
12.		25.	
13.			

5. What was the average number of boxe purchased per trial?
(Average = total boxes ÷ number of trials) . . . .

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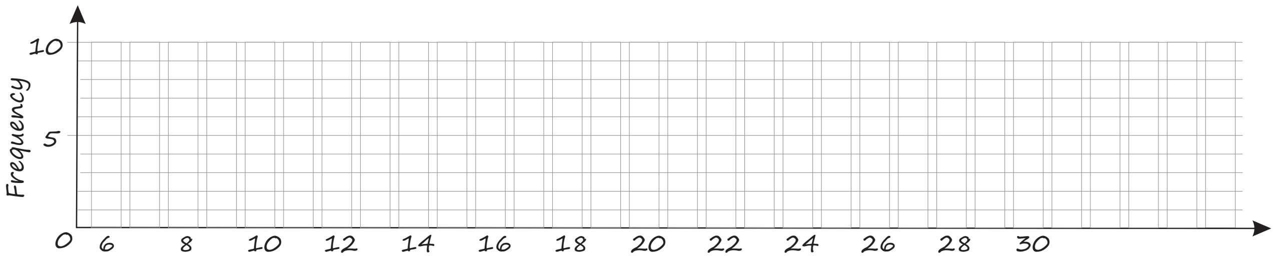
6. Complete the graph below that shows the average number of boxes of tea that need to be purchased in order to receive the tea pot.

Packets of Tea Purchased (25 trials)



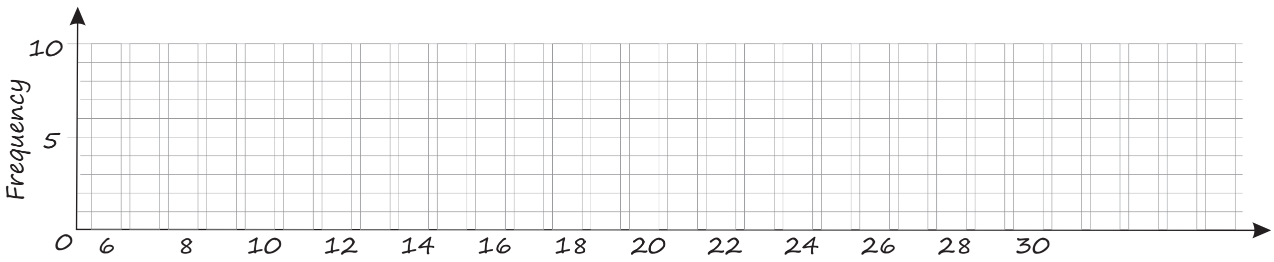
7. If you carried out an additional 25 trials would your results be the same? Explain your answer.

. . . .

. . . .

8. Collect results from 7 other people. Combined with your own results it means that you should have 200 trial results. Complete the graph below that shows the average number of boxes of tea that need to be purchased in order to receive the tea pot.

Packets of Tea Purchased (200 trials)



9. What is the average number of boxes purchased over the 200 trials?

. . . .

10. If there were 8 pieces of a jigsaw puzzles instead of 6, would you need to buy more boxes of tea or fewer boxes? Explain your answer.

. . . .

. . . .

11. Imagine that there was a mixup at the factory and one of the puzzle pieces was put in 25% of the boxes of tea while the other 5 were randomly distributed among the other 75% of the boxes. Design and carry out a new simulation to predict the average number of boxes that you would need to buy in order to get a complete jigsaw set.

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TEACHER NOTES

THE ELEVATOR

At the end of this simulation exercise teachers might want to collect all the combined data onto a master list and then use it later to illustrate central tendency. It is always best to start the situation by drawing a picture of the building on the board and then ask the students for ideas on how to complete a simulation. You could also ask how many different trials would be needed to achieve a successfully result that is close to the theory.

Answers to Questions

1. The simulation can be achieved by a number of methods - Excel or other computer programme, 3 dice, a single die thrown 3 times or by using the RND# tool on a scientific calculator.

2. Each worker can get off on any of the 6 floors. This means that for 3 workers there are $6 \times 6 \times 6 = 6^3$

$= 216$ different possible combinations of floor choices.

3. The first worker can get off on any of the 6 floors. To get off on a different floor the second worker only has 5 choices and the third worker only has 4 choices. This means $6 \times 5 \times 4 = 120$ different possibilities.

4. Using the results from 1 and 2: $120 \div 216 = 0.556$.

5. Answers will vary but as the number of trials gets larger, then the experimental probability should approach the theoretical probability.

THE LIE DETECTOR

This exercise has possibilities as an introduction to conditional probability. Students can work individually or in pairs with 1 pair determining whether the suspect is actually guilty and the other determining whether the lie detector results are accurate or not. As each individual pair or group collects the data the teacher could combine all the results onto a master list. At the end this can also be used to test for central tendency. A good investigative question is to get the class to calculate the chance that a guilty person is found guilty. To do this they need to use the ratio:

$$\frac{\text{number of times test concludes guilty and suspect is actually guilty}}{\text{number of times test concludes guilty}}$$

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SPYDER

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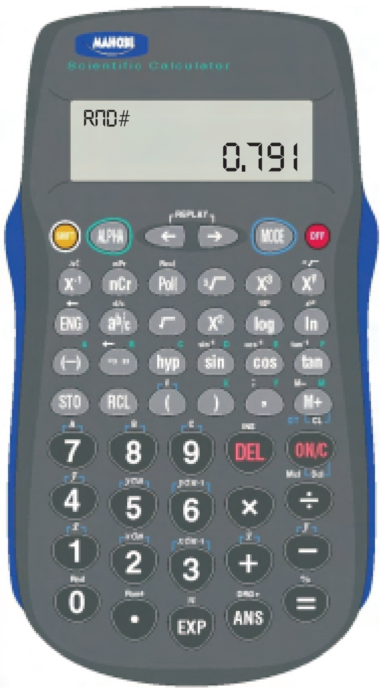

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THE ELEVATOR

Mahobe Towers is a 6 story building with an underground parking garage. One morning 3 workers have parked their cars and are waiting for the elevator. Each worker has an equal probability of getting off at any floor.

Jackie has a theory that it is more probable that each worker will get out of the lift at different floors rather than 2 or 3 of them getting off the lift at the same floor.

Do you think Jackie's theory is right or wrong? . . . .

Design a simulation of 20 trials to test the theory then using the results of 4 other students collect the data for 100 trials.

Individual Data

Trial	Floors	Different	Trial	Floors	Different
e.g.	3 2 4	yes		1 1 6	no
1.			11.		
2.			12.		
3.			13.		
4.			14.		
5.			15.		
6.			16.		
7.			17.		
8.			18.		
9.			19.		
10.			20.		

Group Data

$\frac{\text{total exits at different floors}}{\text{number of trials}}$

	total rides	total exits on different floors	Fraction	Decimal
1.	20			
2.	40			
3.	60			
4.	80			
5.	100			

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1. Write the method you used to simulate 3 workers traveling in a lift and getting out at any one of six floors.

. . . .

. . . .

. . . .

2. If you had to make a list of all the possible combinations of floor choices for each of the 3 workers how many would be on the list? Explain how you calculated this answer.

. . . .

3. From your list of choices, how many would have the workers all getting off on different floors?

. . . .

4. Calculate the theoretical probability that all 3 workers get off the elevator at different floors.

. . . .

5. Compare the experimental results with your theoretical results.

. . . .

. . . .

. . . .

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THE LIE DETECTOR TEST

There has been a jewel theft and the police have three suspects. They arrest one of these suspects however, when questioning her, she says that she had nothing to do with the robbery.

Police decide to give her a Polygraph (lie detector) Test. Research has shown that the test is only accurate 80% of the time. This means that 20% of the time that a person is telling the truth the test will conclude they are lying and 20% of the time a suspect is lying, the test will conclude that they are telling the truth.

This simulation is in two parts. Firstly you need to determine whether the suspect is actually guilty or not guilty. By using the RAN# key on your calculator generate either a 1, 2 or 3. If the number displayed is a 1 then the suspect is guilty. Alternatively you could use a die. If a 1 or 2 is displayed then the suspect is determined as guilty.

In the second part of the simulation you need to decide on the validity of the Polygraph Test. To do this generate a random number between 0 and 1. If the number displayed is less than 0.8 then the test result is accurate.

1. Complete the following table of results.

Trial	Suspect Guilty (G) or Not Guilty (NG)	Test Result Accurate (A) or Not Accurate (NA)	Conclusion from Test Suspect G or NG
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			
11.			
12.			
13.			
14.			
15.			

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RESULT SUMMARY Number ÷ 15

2. Suspect is guilty and Polygraph conclusion is guilty . . . . .

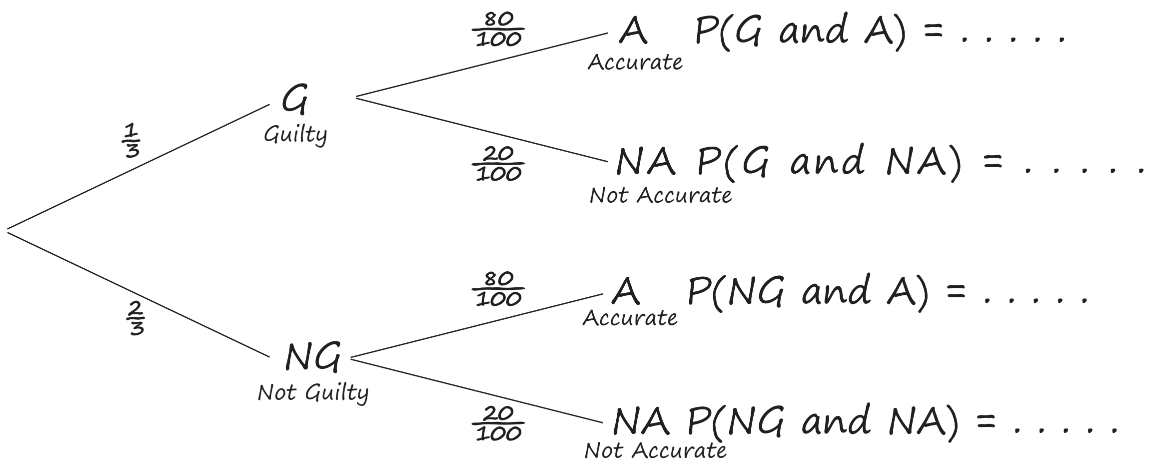
3. Suspect is guilty and Polygraph conclusion is not guilty . . . . .

4. Suspect is not guilty and Polygraph conclusion is guilty . . . . .

5. Suspect is not guilty and Polygraph conclusion is not guilty . . . . .

THEORETICAL PROBABILITY

Below is a probability tree diagram for this simulation.
Calculate each of the concluding test results as decimals.



6. Do you think the Polygraph is a good way of solving a crime?
Give your reasons.

. . . .

. . . .

. . . .

7. If you did the simulation again would you get the same results?

. . . .

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MONTY HALL

The television programme “Lets Make a Deal” was a long running game show that featured a problematic situation. A randomly selected audience member would come onto the stage and be presented with 3 doors. Behind one of the doors was a brand new car but behind the other two were donkeys.

After making their selection the host, Monty Hall, would reveal one of the donkeys behind one of the non-selected doors. This means that there are now two doors unopened - one that the contestant has chosen and another. The contestant is now asked whether they want to stay with their original selection or switch. At this point the suspense would heighten as the rest of the audience would shout out their preference - “stay” or “switch”.

1. Should the contestant stay with their original selection, or switch to the other unopened door. Explain your answer.

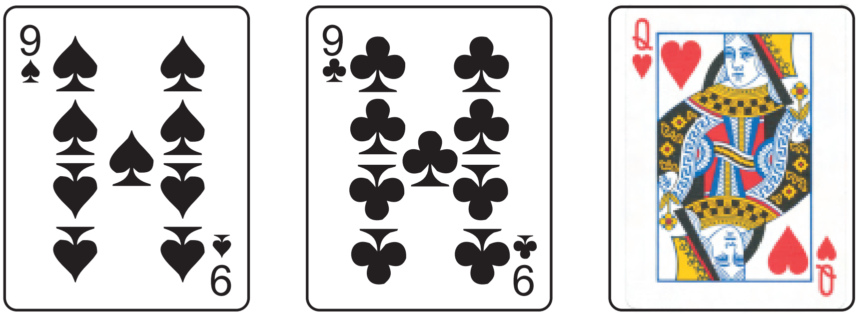
. . . .

. . . .

. . . .

The problem or staying or switching caused many an argument and was also a topic of discussion on the front page of the New York Times. In 1990 Marilyn vos Savant, who was listed in the Guinness Book of World Records for the highest I.Q., argued that you had a much higher probability of winning the car if you switched doors. Her argument provoked letters from thousands of readers, academics and mathematicians who nearly all argued that doors #1, #2 and #3 each had an equal chance of success.

In this simulation you are going to test Marilyn vos Savants argument. With a partner you will be given three cards. Two could be ordinary cards (donkeys) and the other the prize “car” card.



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The STAY simulation

In the first 30 simulations your partner will hold up the cards so that you cannot see the cards' faces. You will then make a selection as to which you think is the prize card. After selecting the card your partner will reveal whether you won the prize or not. Keep a count of how many times you win the prize.

Results of 30 simulations - STAY with your selection

Wins	
Loses	

The SWITCH simulation

In the second 30 simulations you repeat the process. However this time after making your selection your partner will put down one of the "donkey" cards and then reveal the card that you didn't choose. This is the card that you would choose if you had switched. Keep a count of how many times you win the prize.

Results of 30 simulations - SWITCH your original selection

Wins	
Loses	

The Theory

Below are the 3 different possible scenarios.

Scenario	Door 1	Door 2	Door 3
1	car	donkey	donkey
2	donkey	car	donkey
3	donkey	donkey	car

If you choose Door 1 and stay with that choice, your chances of having a car are only 1 in 3. In each of the scenarios Monty Hall has to reveal a donkey. This means that in Scenario 2 and 3 you are better to switch your choice. Only in Scenario 1 would you be better off staying with your original choice. Therefore you have 2 out of 3 chances of winning if your switch doors.

2. On your own paper, write a brief conclusion about your simulation results.

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TEACHER NOTES

RADIOACTIVE DECAY

This activity does need a minimum of 100 dice. These are readily obtained in most $2 or gift shops, usually in packets of 10 or 20 depending on the size of the dice.

The subject of radioactivity is an emotive subject especially in New Zealand which is ‘nuclear free’. There are many dangers posed from radioactivity from both peaceful electricity generation and from nuclear bombs. The lesson can also lead into class discussion of radioactivity and substances such as Plutonium-239 which has a half life of over 24 000 years. Each year an average nuclear reactor produces approximately 200 kg of this as waste byproduct of the electricity generation. One of the dangers is that it is also an ingredient of the atom bomb.

The lesson has potential of going in different directions with discussions on exponential decay, “half life”, the many good and bad uses of radioactive material..

Here are some other investigative questions that could be posed for students:

1. How easy is it to throw a 6? How long on average does it take to throw a 6? Students could run a simulation and graph the results. They could report on the average number of throws and how the results were spread.

2. When simulating the decay of 100 atoms, record how long it takes to reduce the overall total of dice to 50 (half life). Do 4, 5 or more simulations and graph the results.

3. A spreadsheet can be used to quickly run a number of simulations. How many years does it take for 100 atoms to decay? Try a number of simulations and record the results. What is the largest time taken, the least time taken, the average time and the most common time? Graph the results.

4. How long does it take for the very last atom to decay? This question is really the same as Question 1 but it is interesting to see how many students can recognise it.

5. If the 6 sided dice was replaced with a 10 sided dice (or calculator and RND# function), what would happen to the rate of decay? What would happen to the half life? If 100 atoms were used in a simulation with 10 sided dice would it take longer or a shorter time for all to decay? Similarly what would happen if you used a coin instead of dice in the simulation?

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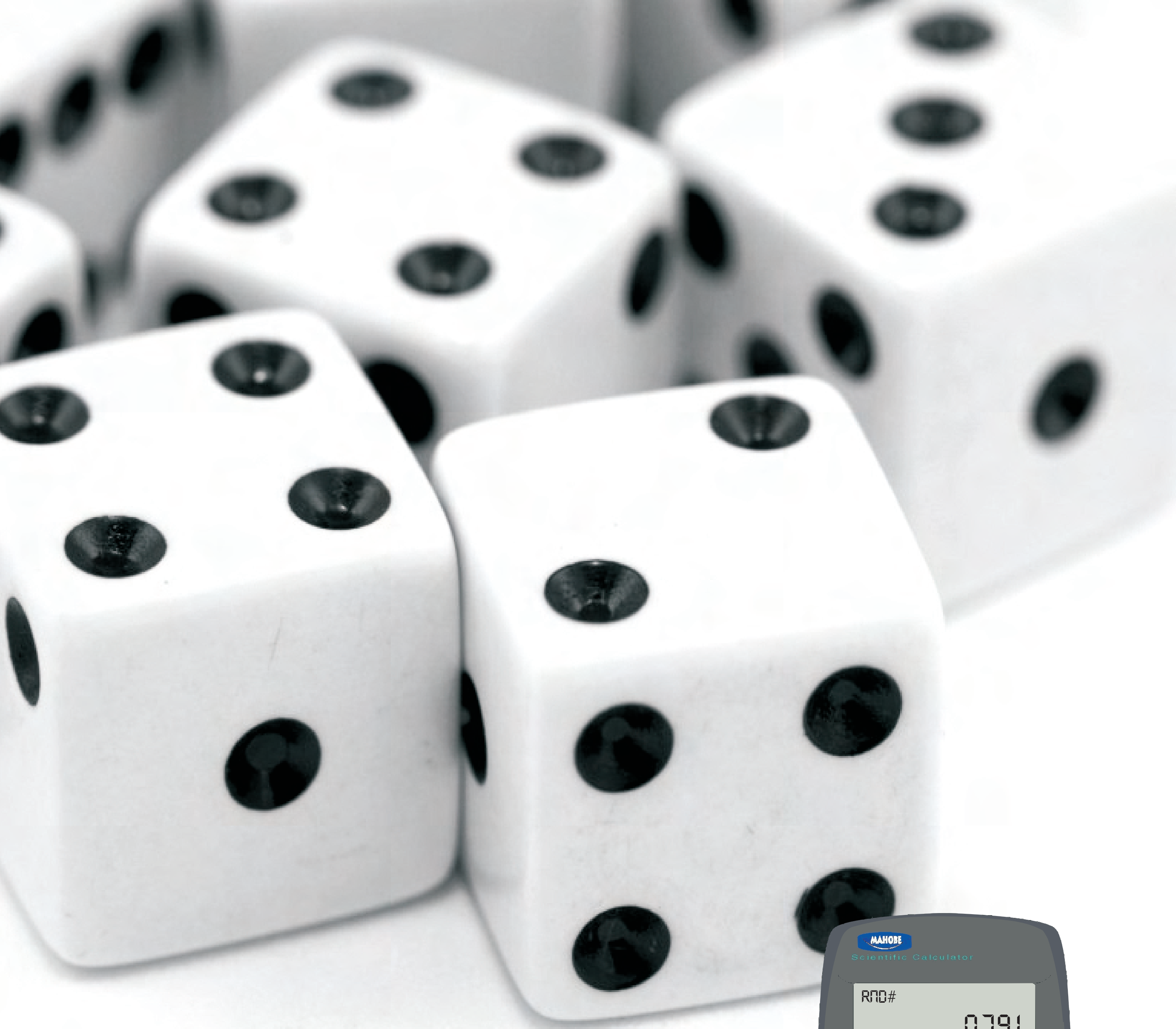



MAHOBE
Scientific Calculator
RND#
0.791

RADIOACTIVE DECAY

Radioactivity describes the process of decay of certain types of atoms. The time it takes for half the initial number of atoms to decay is called the half life of a substance. From that point it takes the same amount of time for the remaining amount to halve. This means that 2 half lives leave a quarter. Three half lives from the beginning leave one eighth and so on. Mathematically the atoms will never all disappear but in practice you can end up with such small amounts that they are not able to be detected.

Introducing Radioactive Decay

1. Each student stands with a die. Throw the die and note the number. The teacher now also throws a die. The students with the same number as the teacher are atoms that have just decayed and they need to sit down. Continue this process until there are no atoms left (nobody is left standing). With each throw record the number. Then complete the graph below.

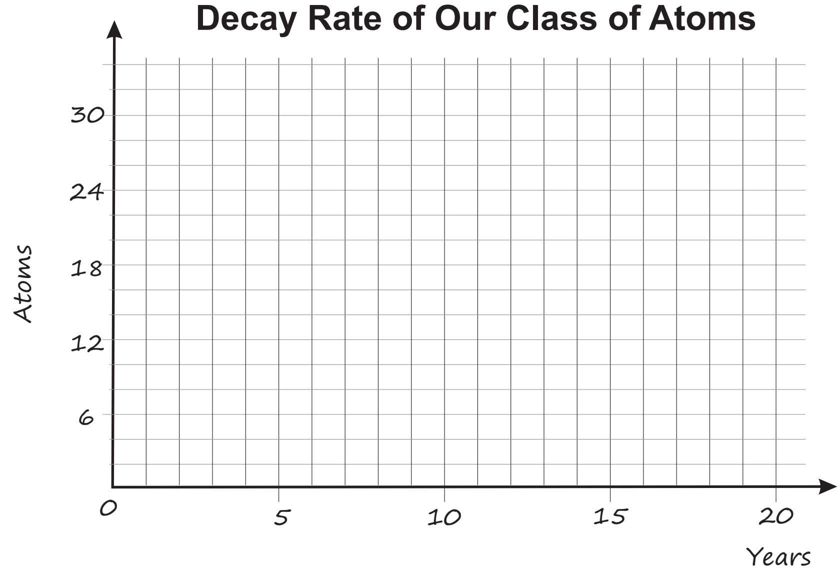
Year	0	1	2	3	4	5	6	7	8	9	10	11	12
Atoms Left													

2. How many rolls did it take for half the class to be sitting? . . . .

3. Did it take approximately the same amount of rolls for the second half life?

. . . .

Decay Rate of Our Class of Atoms



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Simulating the Decay of 100 Atoms

For this simulation you need 100 dice. Throw the dice. Count and take out all those that have a 6 face up. Now throw the remaining dice and repeat the process by counting and taking out all the dice with a 6 face up. Keep on throwing the dice until there are none left. Complete the table below as you go.

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Atoms	100														

5. How many years does it take for the 100 atoms to decay? . . . .

6. Paula has a simple theory that because each atom has a 1 in 6 chance of decaying then it should only take 6 years for all to be gone.

Comment on this theory.

. . . .

. . . .

7. Jones has a theory that as each atom has a 1 in 6 chance of decay then it means that in each year 5 out of 6 atoms are still left. He has drawn up the following calculations.

Year 1: $100 \times \frac{5}{6} = 83$ Year 2: $83 \times \frac{5}{6} = 69$ Year 3: $69 \times \frac{5}{6} = 58$

He has also come up with a formula $T = ar^n$

Where T = the **T**otal amount of atoms left

a = the initial **a**mount of atoms

r = the **r**ate of decay

n = the **n**umber of years

Continue with his calculations and compare them to the simulation results above. Comment on Jones' theory.

. . . .

. . . .

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TEACHER NOTES

BIRTHDAY MATCHES

This activity is similar to a famous birthday paradox that shows in a group of 23 people there is a 50% chance that at least two people will share the same birthday date. In a group of 35 people the probability of two sharing the same date is a little over 80%.

A fun way of initially collecting the data is to have the students in the class walk around the room, much like in the game musical chairs. When the time is up they must grab the nearest people next to them to make a group of 5. If any students in the group have matching birth months then the outcome is considered to be a success.

From question 4 onwards students can work in groups. This makes it simpler and more manageable for classrooms and easier to get a good result. It can help lead to exploring the mathematics involved and then moving on to a mathematical explanation. In the initial section, most students will agree that 5 trials is not enough but the final trial number and the method of their simulation will be different.

To collect more data some students may want to run a survey in the playground by asking sets of 5 random people what their birth months are and recording the results. This would be the slowest method. Other methods to achieve the same results are to have a 12 sided die or use a suit of playing cards from Ace to Queen. Shuffle the pack and then have someone choose a card. Record the number then replace the card, shuffle and repeat. Another similar method is to make up 12 cards with each of the months printed on them. Use the same method as with a the previous instructions. However note that some students may not be good “shufflers” and this method may not produce true randomness. The final methods are to use the RND# function on a calculator or a spreadsheet. A random number method has the most advantages in that it is the quickest and it has true randomness. However in terms of student perception, using a calculator is probably the most boring method. For all the methods given, the advantages and the disadvantages are good discussion points to have with the class.

Some students may note that different months have different lengths. While this doesn’t make much difference to the result at least they are trying to make their simulation as accurate as possible and are potentially “excellence” students in NCEA.

To find the actual theoretical probability it is best to start with a different problem such as “Five people meet, what is the probability that all have different birth months?” The calculations are over the page.

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Person	This means:
Person 1 can be born on any of the 12 months.	12 out of 12 possibilities.
Person 2 can only be born on 11 of the months as they cannot be born on the same month as Person 1.	11 out of 12 possibilities.
Person 3 can only be born on 10 of the months as they cannot be born on the same month as Persons 1 and 2.	10 out of 12 possibilities.
Person 4 can only be born on 9 of the months as they cannot be born on the same month as Persons 1, 2 or 3.	9 out of 12 possibilities.
Person 5 can only be born on 8 of the months as they cannot be born on the same month as Persons 1, 2, 3 or 4.	8 out of 12 possibilities.

$$\frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} = 0.382$$

If there is a 0.382 chance (38.2%) of being different then the other 0.618 (61.8%) must be a match.

Question 8 is designed as an extra project or an extension question. When you vary the size of the group then the probability calculations can be worked out with the same method. The table below gives all the various probabilities:

Group size	4	5	6	7	8	9	10	11	12	13
Probability	0.427	0.618	0.777	0.888	0.954	0.985	0.996	0.9994	0.9999	1.0

The final question - The Birthday Paradox

There is a class of 23 students. What is the probability that 2 of them will have exactly the same birth date? The theoretical logic is the same as above. Calculate the probability that there is no match.

i.e Probability of no match is: $\frac{365 \times 364 \times 363 \times ..... \times 343}{365 \times 365 \times 365 \times ..... \times 365} = 0.5$

This means the probability of a match is 50%

Similarly the probability of a birthday date match in a group of 35 people is 81.4%. Interestingly there have been 40 Prime Ministers of New Zealand since 1856. A good maths project would be for students to calculate the theoretical probability of a match (89.1%) then research all the dates of birth. Are there 2 with the same birth date? A similar random group would be a random selection of teachers in a school.

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BIRTHDAY MATCHES

1. If you chose 5 people in your class, what do you think the probability is that at least 2 of them will have their birthdays in the same month?

Mark on the line below what your estimate would be.

not much chance	50/50	a certain chance
0	0.5	1

2. Just having one trial in any simulation will not give you a very conclusive result. Therefore fill in the grid below with 5 groups of 5 peoples birthday months. Each complete group of 5 will be one trial.

	Months					Success?
Sample:	5	7	6	6	1	✔
Trial 1.						
2.						
3.						
4.						
5.						

3. Calculate the percentage of trials that had 2 or more people with birthdays on the same month.

. . . .

4. Zelma does not think that 5 trials are enough to calculate the true probability. She thinks it would be better to question random students at lunch time and record a much larger number of trials. Is this a good method? Write below any others ways you could run a simulation and write what would be an ideal number of trials to get a good estimate of the probability of at least 2 people in a group of 5 having the same birth month.

. . . .

. . . .

. . . .

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Simulating the Chance of a Birthday Match

5. Carry out a simulation to calculate the probability of at least 2 people in a group of 5 having a birthday in the same month. Write your method and your conclusions below.

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

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6. Can you calculate the actual theoretical probability of at least two people in a group of 5 having the same birth month?

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7. How close was your estimate from question 1?

. . . .

8. What would happen to the probability of 2 people in a group having the same birth month if the number in the group gets larger (e.g. 6, 7 or more people)? Carry out a simulation and write a report on your method and your results.

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TEACHER NOTES

THE SPREAD OF DISEASE

This activity requires every student to have a die. Teachers may want to begin the simulation with a discussion about infectious diseases such as colds, the flu, or chicken pox. How certain is it that a person will become ill if they are exposed to someone who is sick? What is the possibility that the person will not catch the disease? What variables might influence a person's resistance to the disease? What factors might make a person more vulnerable to the disease?

Medical professionals look to mathematics and statistics to determine the likelihood of infection of different diseases and the rate at which any disease might spread through a population.

The first part of the simulation requires students to walk around and "encounter" other students. With each encounter they throw their die and record the name of the person and the sum of the two dice. Within the simulation they should discover that certain sums will occur more often than others.

The answers to the questions will vary with the class and the results. The graph in Question 8 will always be increasing but will eventually level off when the majority of students become infected. Within the lesson there can be discussion around issues or circumstances such as:

Some diseases are more contagious than others. How would that affect any graph of the infections?

What would happen if instead of 5 stages there were 25 stages. Would everybody in the class become infected?

Will everybody eventually become infected if there were unlimited stages?

What would happen if there was a larger sample or even an unlimited number of people exposed to this flu? How would that effect the results?

The whole simulation can also be rerun using the same data but using a different initial "infectious" person or by changing the sum of the two dice.

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THE SPREAD OF DISEASE

What can cause a disease to spread from one person to another? If an infected person breathes or sneezes on you, how likely is it that you will become infected? Will you definitely become infected or could you possibly remain uninfected?

1. This activity simulates the spread of a new flu virus at your school. To simulate the outbreak you are going to move around the room for 3 minutes and record all the other students that you come into contact with. As you meet each student, record their name in the table below. With each contact roll your dice and record the sum. After 3 minutes your teacher will instruct you to move to Stage 2 on the table, record the names and the sums. Keep on repeating this until you have completed 5 stages of 3 minutes.

Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Name, Sum	Name, Sum	Name, Sum	Name, Sum	Name, Sum

2. Now that you have now recorded all the encounters, your teacher will randomly select a name from the roll. This is the person that brought the virus into your class. The teacher will also roll two dice. If you had an encounter with that person and rolled the same sum then you now have the flu. Circle all encounters with that person and that sum.

Your teacher will keep a master list on the board of all those students that are infected and the stage at which they were infected.

3. For stage 2 onwards circle any encounter that you had with newly infected people if you had the same “contagious” sum.

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4. Was this a contagious disease? Explain your answer.

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5. When you throw two dice, certain sums are more likely to occur than others. Below is the theoretical matrix of sums. Use the matrix to determine the theoretical probability of each sum.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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6. When you came into contact with a contagious person, did you always become infected? How many times did you come into contact with a contagious person and not become infected? What made the difference?

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7. Complete the table below. It shows the infection rate of people in your class.

Stage Number	Number of New People Infected	Total Infections
0		1
1		
2		
3		
4		
5		

8. If you had more stages and you graphed the total infections at each stage, what pattern would the graph show?

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TEACHER NOTES

AFTER EWE

This activity revolves around sheep breeding and a flock of 1500 sheep. Using given probabilities, students simulate the possible numbers of lambs being produced over three seasons. This activity leads to theoretical probability and tree diagrams but is also a good introduction to normal distribution. There is also the possibility of introducing conditional probability with such questions as “What is the probability that a ewe will have twin lambs given that it has already had twin lambs?”

This activity is more suited for the RAN# function on a calculator but could also be carried out using a spreadsheet or random number tables. If using a computer or spreadsheet simulation then a random numbers simulation could easily be used to get simulated data for all 1500 sheep. However the exercise can also lead to discussion on what would be an appropriate number of trials, could results be combined with other students etc.

When students later perform such simulations for NCEA they are given a situation and are expected to be able to describe a possible simulation scenario. They are then expected to complete a simulation using their own method. Merit answers usually revolve around the calculations of the simulation and theoretical means and probability. Excellence questions usually require them to state the limitations of the simulation and / or possible improvements to the model.

This simulation model assumes that the probability of lambs being born remains constant over each of the three years. In reality there are other external factors such as the weather conditions which can mean insufficient grass or feed, overly cold lambing seasons etc. This in turn can effect the health of the ewes and therefore fertility. Some students from farming backgrounds will also argue about the given birth rates and the fact that some ewes can produce triplets. In such cases the whole scenario can be discussed at the beginning of the lesson and the whole activity can then be run using the student generated probabilities and even by using stock numbers from a farm in the area. For these students the simulation now becomes a lot more realistic.

There will be some students who argue that conducting only 50 trials is not enough when there are 1500 ewes in the herd. This equates to only 3% of the herd. However these students need to be reminded of the purpose of a random sample and why simulations are carried out.

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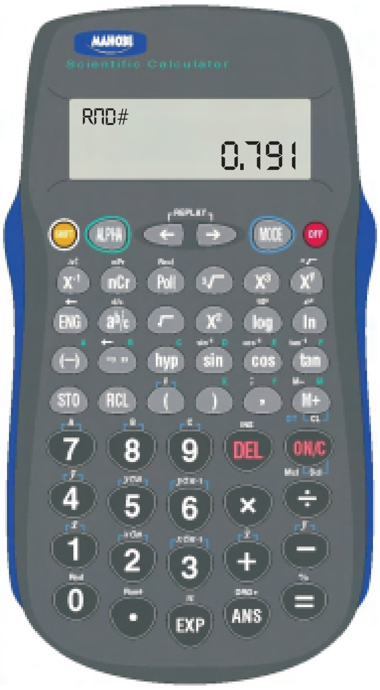

SPYDER

If buying class sets of calculators then this is the logical choice. They are easily recognisable but more importantly they are tough, reliable and contain all the functions that students need. Only the Mahobe SPYDER calculator is recommended by The New Zealand Centre of Mathematics. Sales of this calculator help bring you outstanding classroom resources. Purchase it direct from the Mahobe website:

www.mahobe.co.nz.

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AFTER EWE

Farmer Boyce has a flock of 1500 ewes. He has asked you to calculate the possible numbers of lambs that will be bred over the next three seasons. Each ewe can either produce a single lamb, twin lambs or in some cases no lambs. From records of past breeding seasons Farmer Boyce has calculated that the probability of a breeding ewe producing a single lamb is 0.75. The probability of twins is 0.2 and the probability of a ewe having no lamb in a season is 0.05.

To carry out this simulation you are going to conduct 50 trials. This simulates a random selection of 50 ewes from the flock of 1500. Use the RAN# function of the calculator. If the random number generated is less than or equal to 0.75 then that ewe will have 1 single lamb. If the random number generated is between 0.75 and up to 0.95 then that ewe will have twin lambs and if the random number generated is greater than 0.95 then that ewe has no lambs, that season.

1. For each of the randomly selected ewes generate 3 random numbers to simulate the 3 seasons. Complete the simulation and on the next page fill out the table with the results.

Answer these questions in your own book or on a separate piece of paper.

2. What is the total number of lambs produced by the ewes over the three seasons?

3. What is the mean number of lambs produced by the ewe over the three seasons?

4. Using the results of your simulation estimate the number of lambs that Farmer Boyce could expect after 3 seasons.

5. Estimate from your simulation the number of ewes in the flock that will have at least 1 set of twins over the 3 seasons.

6. Farmer Boyce realises that every sheep has to earn its way. If a ewe does not produce at least 3 lambs in 3 seasons then it is sent to the abattoir. Estimate the number of ewes that will be sent to the abattoir after 3 years.

7. Draw a probability tree and calculate the different theoretical probabilities of a ewe having 0 to 9 lambs over the 3 seasons. Use these probabilities to comment on the results of your simulation.

8. What are the limitations of this model for predicting the number of lambs produced by Farmer Boyce's flock over several years? Give at least two limitations.

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AFTER EWE - SIMULATION RESULTS

	Seasons					Seasons			
Trial	1	2	3	Total	Trial	1	2	3	Total
1.	....	....	....	....	26.	....	....	....	....
2.	....	....	....	....	27.	....	....	....	....
3.	....	....	....	....	28.	....	....	....	....
4	.....	....	....	....	29.	....	....	....	....
5.	....	....	....	....	30.	....	....	....	....
6.	....	....	....	....	31.	....	....	....	....
7.	....	....	....	....	32.	....	....	....	....
8.	....	....	....	....	33.	....	....	....	....
9.	....	....	....	....	34.	....	....	....	....
10.	....	....	....	....	35.	....	....	....	....
11.	....	....	....	....	36.	....	....	....	....
12.	....	....	....	....	37.	....	....	....	....
13.	....	....	....	....	38.	....	....	....	....
14.	....	....	....	....	39.	....	....	....	....
15.	....	....	....	....	40.	....	....	....	....
16.	....	....	....	....	41.	....	....	....	....
17.	....	....	....	....	42.	....	....	....	....
18.	....	....	....	....	43.	....	....	....	....
19.	....	....	....	....	44.	....	....	....	....
20.	....	....	....	....	45.	....	....	....	....
21.	....	....	....	....	46.	....	....	....	....
22.	....	....	....	....	47.	....	....	....	....
23.	....	....	....	....	48.	....	....	....	....
24.	....	....	....	....	49.	....	....	....	....
25.	....	....	....	....	50.	....	....	....	....
			TOTAL	....				TOTAL	....

GRAND TOTAL ....

SIMULATIONS CHANCE AND DATA, by Kim Freeman
ISBN 9781877489259

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TEACHER NOTES

AIRLINE SEATS

This activity was inspired after reading an article in Fortune Magazine. The article covered airline bookings and indicated that some flights were regularly over booked, some by as high as 150 - 200%. It also referred to a group of people who booked a number of flights that they had no intention of traveling on. They would then turn up late for the flight in the most likely hope that it had been over booked and that they could then claim a travel voucher. In the USA, being forced to take a different flight is against the law, and therefore airlines have to give monetary vouchers to those who cannot be placed on a booked flight. There are of course severe penalties for those who try and cheat the system.

Airlines, like most businesses, have to make money to survive. The best way to do this is to ensure that there are as few empty seats on their planes as possible. When a person doesn't show up for a flight, their seat is empty and an empty seat is lost revenue. As a result of this airlines have employed statisticians to figure out how many seats the airline needs to over book just to make up for the no-shows. They have also calculated that it is a better deal for an airline to give out a $300 travel voucher and a free seat on another plane than it is to risk letting a seat fly empty.

This is an enjoyable exercise that can lead off to discussion of airline flights and holiday destinations. Some students will have experienced scenarios where they have seemingly arrived on time for a flight but have been told that their seat was taken. Sometimes with destinations that require more than 1 plane this can be common. Holiday times are the worst for over booking especially if weather conditions are bad at either the departure or arrival destinations. Therefore you will find that at least one student has experienced this.

Students should be familiar with the RAN# key of their calculators. To simulate an 80% - 20% scenario then any random number up to 0.8 is considered to be a 1 (passenger turns up) and any random number greater than 0.8 is considered to be a 0 (passenger does not show up). You could also run the simulation with a 10 sided die or a spreadsheet.

In the final questions students have to imagine that they are the owner of the airline. Do they over book and then run the risk of spoiling peoples' holidays or do they try and make the maximum profit possible?

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AIRLINE SEATS

Most airlines will over book their flights, sometimes by as much as 50%. This is because not everyone who has booked will actually turn up for their flight. There are various reasons for this such as illness, arriving early and flying out on an earlier flight or travelers making duplicate bookings on multiple flights so that they have flexibility to travel on a flight that is more convenient.

Airlines know that not every booking for any flight will result in the person turning up to travel. By keeping historical records of past flights, and matching it to other variables such as weather at the departure and arrival points, holidays periods and other flight cancellations they have developed good forecasting models that give guidelines for the percentage of travelers booked on a flight who will actually travel and not travel. This information allows them to accept a number of over bookings with equanimity, and usually with few problems.

In this activity you will explore the number of seats that need to be sold in order to fill a 20 seat plane. Historical information indicates that 20% of passengers do not turn up for their flight.

Answer these questions in your own book or on a separate piece of paper.

1. Using the RAN# function on your calculator, how can you simulate whether each passenger will show up or not show up for a flight?

2. Complete the table on the next page that shows 4 flights of a 20 seat plane. Use a 1 if they showed for the flight and a 0 if they did not.

3. In your 4 flight simulation, how many people on average did not show up for the flight?

4. How many students in your class had at least 1 full flight?

5. On average how many seats were empty on each flight?

6. How many seats do you think you might need to sell to ensure the flight is full each time? Explain.

7. Run your simulation again but this time add the extra seats you recommended in question 6. Discuss your results.

8. During one particularly bad period the no-show rate increases to 30%? How many seats would then need to be sold to ensure the flights are full each time? For costs to break even the plane has to carry at least 15 people. If you were the owner of the company, and wanted to make the most profit possible, how many over bookings would you instruct your staff to take. Explain your answer.

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AIRLINE SEATS- SIMULATION RESULTS

	Flights			
Passenger	1	2	3	4
1.	....	....	....	....
2.	....	....	....	....
3.	....	....	....	....
4	.....	....	....	....
5.	....	....	....	....
6.	....	....	....	....
7.	....	....	....	....
8.	....	....	....	....
9.	....	....	....	....
10.	....	....	....	....
11.	....	....	....	....
12.	....	....	....	....
13.	....	....	....	....
14.	....	....	....	....
15.	....	....	....	....
16.	....	....	....	....
17.	....	....	....	....
18.	....	....	....	....
19.	....	....	....	....
20.	....	....	....	....

	Flights			
Passenger	1	2	3	4
1.	....	....	....	....
2.	....	....	....	....
3.	....	....	....	....
4.	....	....	....	....
5.	....	....	....	....
6.	....	....	....	....
7.	....	....	....	....
8.	....	....	....	....
9.	....	....	....	....
10.	....	....	....	....
11.	....	....	....	....
12.	....	....	....	....
13.	....	....	....	....
14.	....	....	....	....
15.	....	....	....	....
16.	....	....	....	....
17.	....	....	....	....
18.	....	....	....	....
19.	....	....	....	....
20.	....	....	....	....
21.	....	....	....	....
22.	....	....	....	....
23.	....	....	....	....
24.	....	....	....	....
25.	....	....	....	....
26.	....	....	....	....
27.	....	....	....	....
28.	....	....	....	....

SIMULATIONS CHANCE AND DATA, by Kim Freeman
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From the author.
Over the past 34 years (and from memory 4 New Zealand maths curriculums) I have used a number of simulations in class to illustrate different situations. This booklet contains most of my favourites. However there are a number of other great possibilities that I have also used.

Coloured Jelly Beans
In this scenario I have a bowl filled with 100 jelly beans comprising equal amounts of 5 colours. The question to students is: "Do I have a better chance of getting the black jelly beans if I select the beans at random until all beans are eaten, or do I have a better chance if I replace each colour as I eat one?" Another twist can be added by asking "If the bean is replaced by a random colour what happens to my chances?"

Jet Boat Rides
After experiencing a jet boat ride one holidays I realised that the scenario also had potential for a class simulation. The jet boat held 6 people but the operators did not separate families and were good enough to let the boat depart even if it was not completely full. Tourists were booked as they arrived. If, for example, two adults, then two adults and 1 child arrived and then finally two adults booked then the last group would have to wait 20 minutes and the jet boat would depart with only 5 people. There are of course lots of scenarios and probabilities to consider so a simulation that worked out a days, weekends or a week's possible income would be a good exercise.

Warrant of Fitness and Car Registration.
Most of us have all been stopped at the roadside and been breath tested and checked for licence, WOF and car registration. It is a useful class statistics exercise for students to investigate the probability of a car having a current WOF and registration. This can be done at the local supermarket, mall car-park or even the teachers' car-park! Then run a simulation to calculate how many the police are likely to catch in a random checkpoint. Is it worth their time?

The Multiple Choice Test.
We have all heard stories and in some cases witnessed students who have sat a multi-choice test and managed to score way above their potential. Is it possible? Design a simulation of a 30 question multiple choice test in which students always choose option B. In another scenario get them to making a random choice each time. What are the chances of passing the test? You will be surprised at some of the results.

Sports
If you are a rugby or basketball fan then there are lots of possible simulation exercises. What is the strike rate for penalties and conversions from your favourite kicker? What are famous basketball players percentages for free throw shots? Rugby conversions have an extra difficulty factor that can be discussed as possible limitations to the simulation and although basket ball players seem to all have the same variables when shooting from the free throw line there are a similar set of limitations with home game /away game scenarios and tiredness factors when comparing the time in the game when each throw is taken.

Yahtzee
The game of Yahtzee involves 5 dice and players score according to the combinations of dice that they throw. At NCEA Level 1 students can be presented with the 5 dice and given an exercise where they have to pose their own investigative question and then perform an experiment to answer their question. They usually come up with questions such as: "I wonder how many combinations of sixes I will get when rolling 5 dice", "I wonder what the chances are of throwing four of a kind in Yahtzee" or "I wonder what the probability is of throwing a pair when I throw 5 dice." Excellence answers in this simulation are achieved when the student can relate the theoretical probability to the experimental simulation probabilities. They also need to be able to present the data in more than one way and explain any patterns in the distribution.

ISBN 9781877489259

SIMULATIONS CHANCE and DATA

by Kim Freeman

A good simulation should define the situation, how the results are to be measured and the tools being used. After carrying out a number of trials and analysing the data there should be a statement of conclusion. This booklet guides students through a number of simulation exercises to show that sometimes in real life, situations don't always come out the way we expect.

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